Disease Mapping: Posterior Probability in the Analysis of Hospital Effects

Spatial Variation in Austrian Health Care Supply

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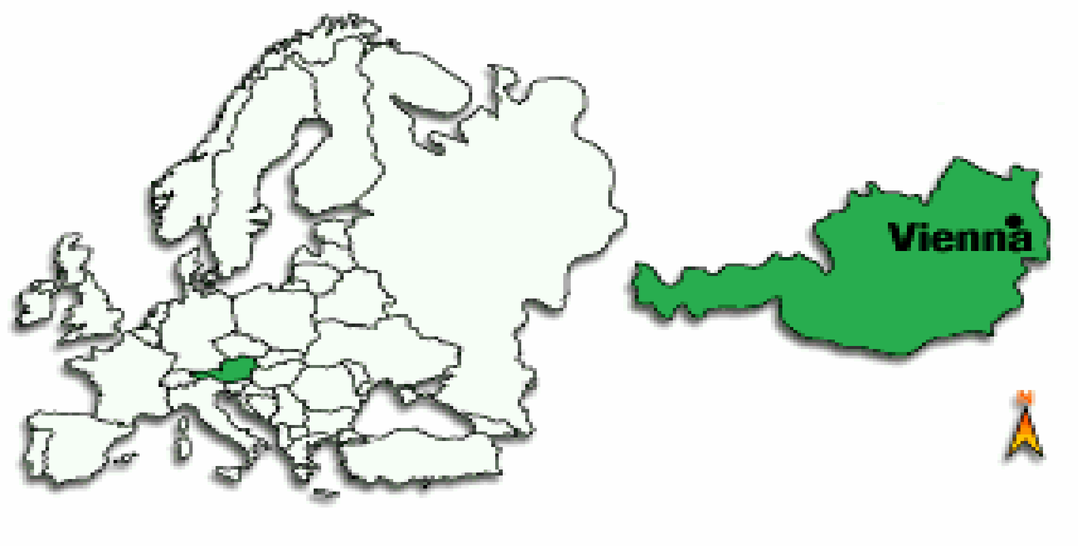
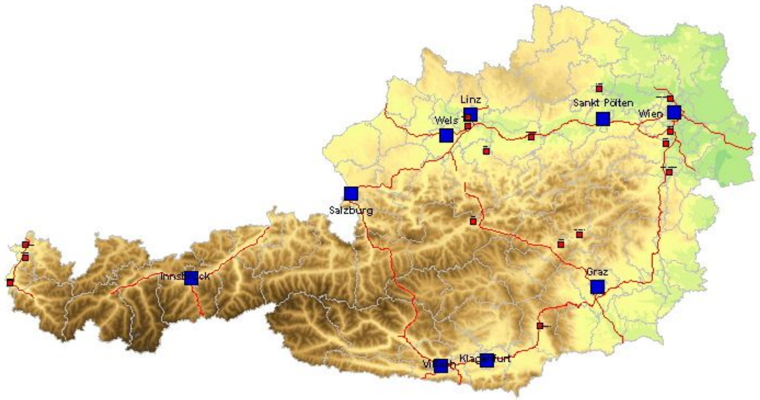
Overview

- The Austrian health care situation
- Why hospital effects?
- Methodological Approach
- Results
- Discussion

Austria



~8,000,000 inhabitants

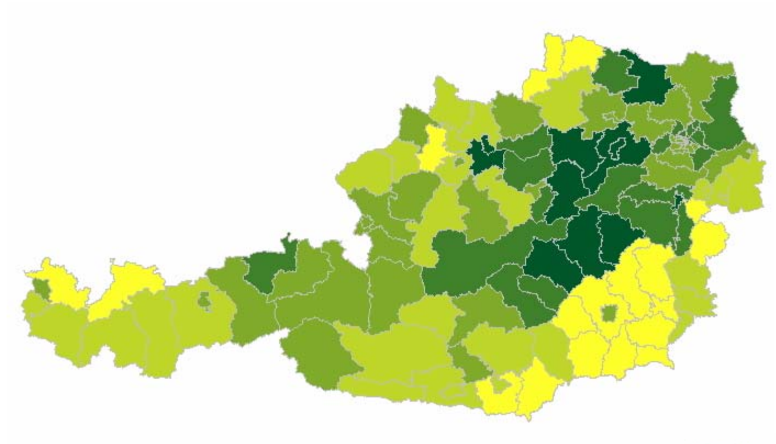


Alpine and flat regions

121 districts

Health Care Situation in Austria

Age and gender standardized hospitalisation ratios (SHRs) for cataract surgery

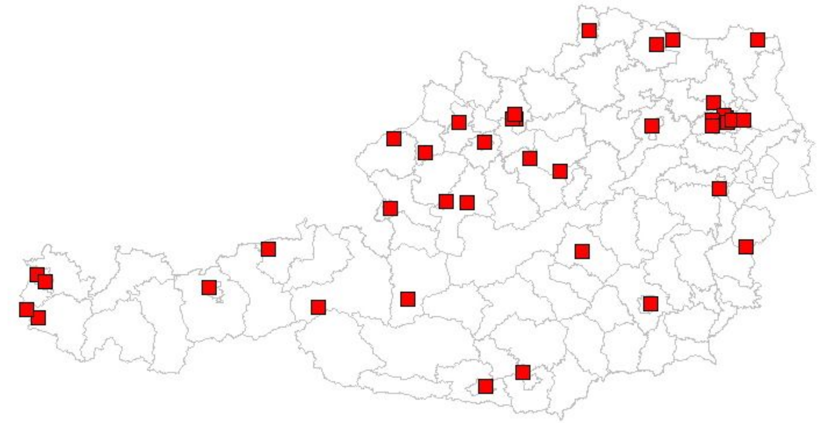


$$SHR_i = \frac{O_i}{E_i}$$

O_i = observed counts in district i

E_i = expected counts in district i assuming the same population structure as in whole Austria

43 hospitals with department of ophthalmology and optometry



SHR_i… estimate for Relative Risk (RR) to get hospitalized in district i compared to whole Austria

Aim

- To model Austrian health care situation
  - Statistically
  - Graphically
- Explain spatial variability between districts by hospitals
  - Same hospitals – similar RR?
  - Same availability – similar RR?
- Need: District hospital interaction model
  - Investigations based on routine hospital data(~2,270,000 hospital admissions / year)
  - Depending on distance, capacity and population size

Bayesian Modelling of Spatial Dependencies

	Pros	Cons
Cluster Model	Clusters around point sources	No interaction modelled
Conditional AutoRegressive model (CAR model)	Neighbourhood interaction regarded Spatial structured and unstructured heterogeneity	Not a surrogate for district hospital interaction
Gravity Model Multivariate model	All hospital- district interaction regarded Patient flows	To many parameters To small sample sizes Graphical presentation?

Ecologic Regression

- Inclusion of one or more hospitals as fixed effects
  - Does not reflect health care situation
  - No interaction regarded
- Fixed effect term
  - averages across all hospitals for each district
- Regard all hospitals in all districts?
  - Definition of a threshold (admission proportion=5%)

But: Hospital effects found to be important

- Suggestion: Use advantages of different models

Availability Model

- Same hospitals – similar RR
  - CAR model with percentages as weights for spatial autocorrelation
- Same availability - similar RR
  - Structured multivariate Gaussian effects with covariances depending on availability
- **Term of structured district-hospital effects**
  - Between hospital interaction
  - Readdress district hospital interaction into patient flows between districts
  - Describe patient flows by n x n matrix Δ – availability matrix

Requirements

- Model for structured area – hospital interaction

$O_i \sim pois(\theta_i E_i)$

O_i...observed counts, E_i...expected counts

θ_i...relative risk in district i

$\log(\theta_i) = \alpha + h_i + \eta_i$

$\alpha \sim flat(-\infty, \infty)$...intercept

$h_i \sim normal(0, \tau^2)$...spatial unstructured effect

η...term for structured area hospital effects

- Is it possible to use a prior like the Conditional AutoRegressive (CAR) prior?

$\eta_i \mid \eta_{j \neq i} \sim N\left(\bar{\mu}_i, \bar{\sigma}_i\right)$

$$\bar{\mu}_i = \frac{\sum_j w_{ij} \eta_t}{\sum_j w_{ij}}$$

$$\bar{\sigma}_i = \frac{\sigma^2}{\sum_j w_{ij}}$$

σ...overall variance, hyperprior assumed

w_{ij}...spatial weights

Conditional Probabilities

- How does joint distribution of prior η look like?

$$\eta \sim N(\mu,\Sigma)$$

$$\mu = (\mu_1,..,\mu_n), i = 1,..,n$$

$$\Sigma_{ii}^{-1} = \frac{\sigma^2}{\sum_j w_{ij}}$$

- $\eta \sim N(\mu,\Sigma)$
  - How is Σ defined?
  - Proper posterior?
- Properties of $\Lambda=\Sigma^{-1}$
  - Symmetry
  - Rank
  - Positive definite / semi definite

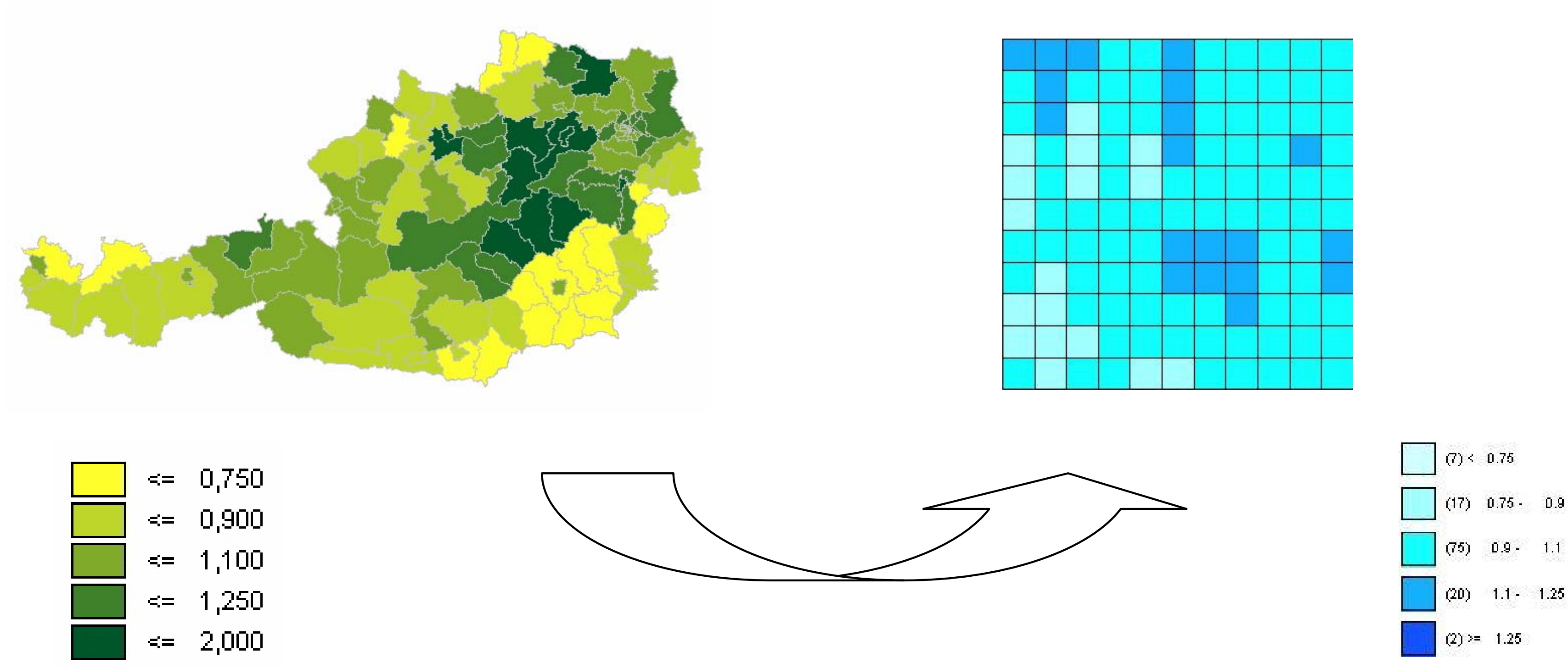
Comparison with CAR model

Δ... Availability matrix new model	Δ …Distance matrix CAR model
Δ not symmetric	Δ symmetric
Δ_{ij} ... Availability weights A... Diagonal matrix with row sums of Δ B diagonal matrix with column sums of Δ	$\Delta_{ij} = w_{ij}$ ...spatial weights
Σ^{-1} full rank or not full rank?	Σ^{-1} weakly diagonal dominant
Some diagonal elements $\Delta_{ii} \neq 0$	Diagonal elements $\Delta_{ii} = 0$
Set $\Delta_{ii}=0$ and $\eta_i \mid \eta_i \sim N(\mu_i, \sigma_i^2)$ with M1 $\mu_i = \alpha\Delta_{ii} + \frac{1}{\frac{1}{2}(A_{ii}^2 + B_{ii}^2)} \sum_{j \neq i} \Delta_{ij}\eta_j$ $\sigma_i = \frac{2\sigma}{A_i^2 + B_i^2}$ M2 $\mu_i = \alpha\Delta_{ii} + \frac{1}{A_{ii}B_{ii}} \sum_{j \neq i} \Delta_{ij}\eta_j$ $\sigma_i = \frac{\sigma}{A_{ii}B_{ii}}$	$\eta_i \mid \eta_i \sim N(\mu_i, \sigma_i^2)$ with CAR $\mu_i = \frac{\sum_t w_{it}\eta_t}{\sum_t w_{it}}$ $\sigma_i = \frac{\sigma^2}{\sum_t w_{it}}$

Simulation

121 districts

121 squares



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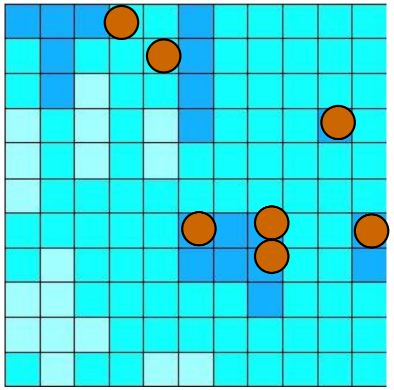
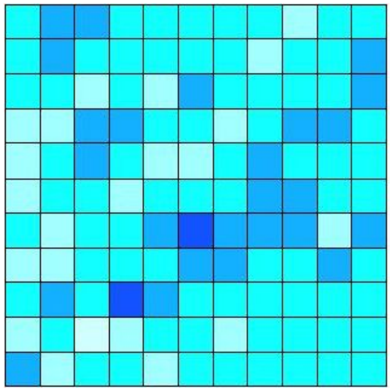
Results- Simulation

True risk

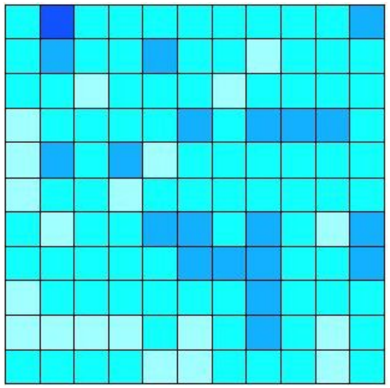


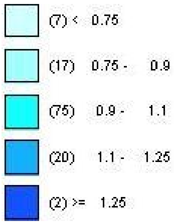
Mean of posterior RR by M1



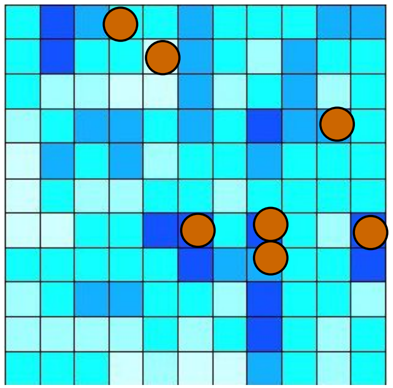
All hospitals available in all districts

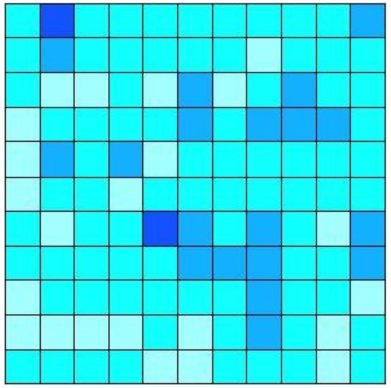
Mean of posterior RR by M2



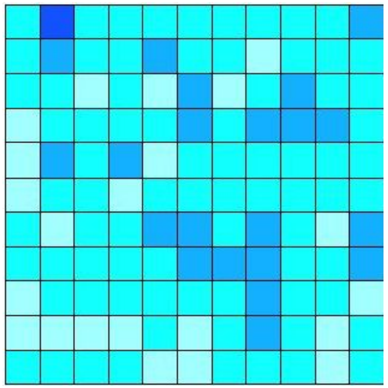


SHR





Availability defined



Notes and Further Work

- Application to real data
- Investigations on distance function
- More than one hospital in a district → average across these hospitals
- Should other parameters be considered?
- Posterior distribution is proper (M2)

Conclusion and Discussion

- Availability model regards between hospital interactions as well as hospital district flows
- Computational time does not depend on the number of hospitals
- Not more parameters than in CAR model
- Symmetric solution for a non symmetric problem
- Variance depends on availability of hospitals
  - Good availability – small variance
  - Weak availability – high variance
  - Within district patient flows not regarded in the variance

Thank you for your Attention!

References:

1. Clayton, D.; Caldor, J. Empirical Bayes estimates of age-standardized relative risks for use in disease mapping. Biometrics 1987,43,671-681.
2. Besag, J.; York, J.; Mollie A. Bayesian Image Restoration, with two Applications in Spatial Statistics Ann. Inst. Statist. Math. 1991,43,1,1- 59.
3. Knorr-Held, L.; Rasser, G. Bayesian Detection of Clusters and Discontinuities in Disease Maps. Biometrics 2000, 56, 13-21.
4. Congdon,P.; Best, N. Small Area Variation in Hospital Admission Rates: Bayesian Adjustment for Primary Care and hospital Factors. Appl. Statistics 2000, 49, 2, 207-226.
5. Curtis, S.; Copeland, A.; Fagg, J.; Congdon, P.; Almong, M.; Fitzpatrick, J. The ecological relationship between deprivation, social isolation and rates of hospital admission for acute psychiatric care: a comparison of London and New York City.Health & Place 2006,12,19-37

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