Disease Mapping: The Effect of Hospitals Availability

Models and Methods

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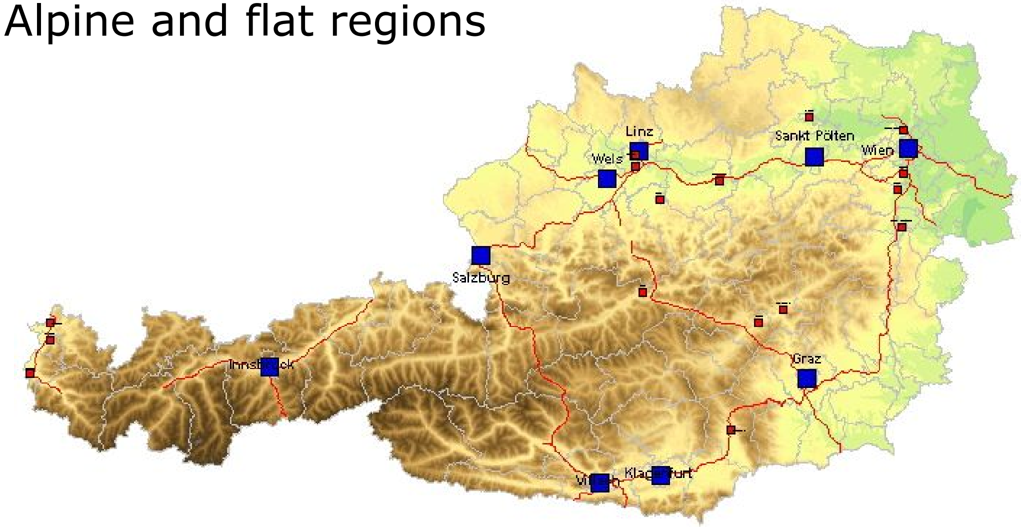
Aim

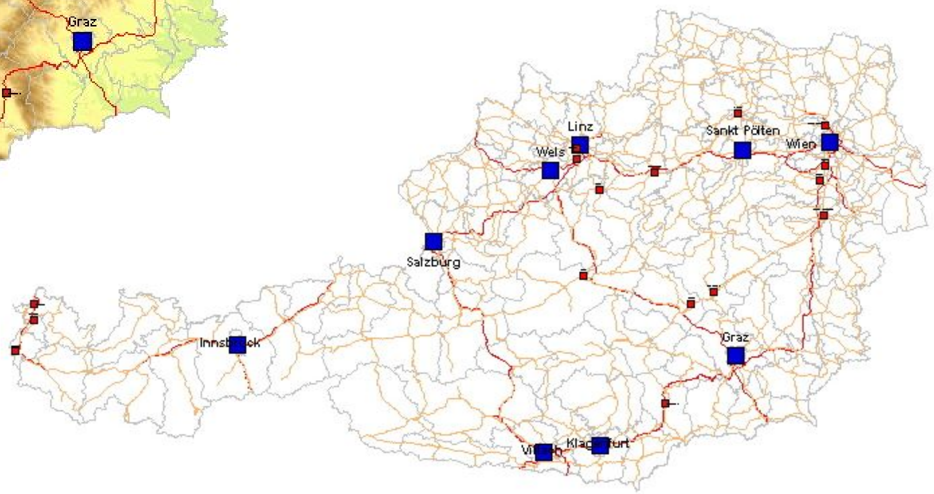
- **To describe –statistically and graphically – the situation of Austrian health care supply**

Practical Relevance – Geographical Situation in Austria

Alpine and flat regions

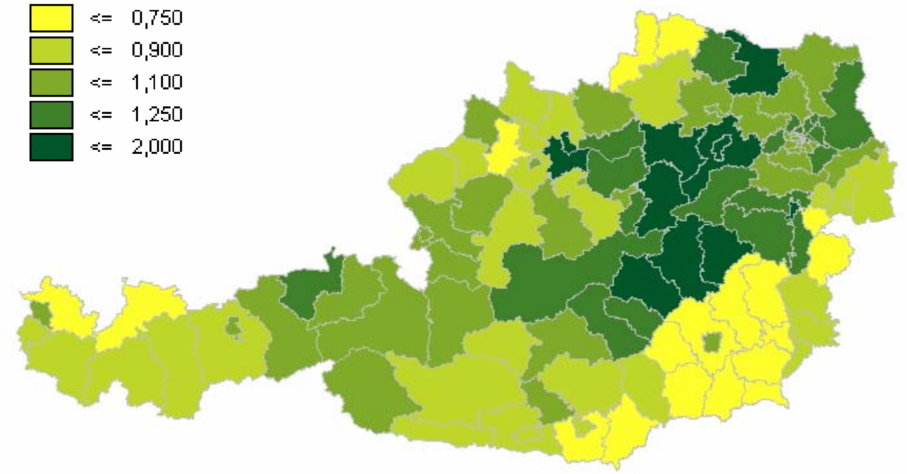


Cities and Traffic routes

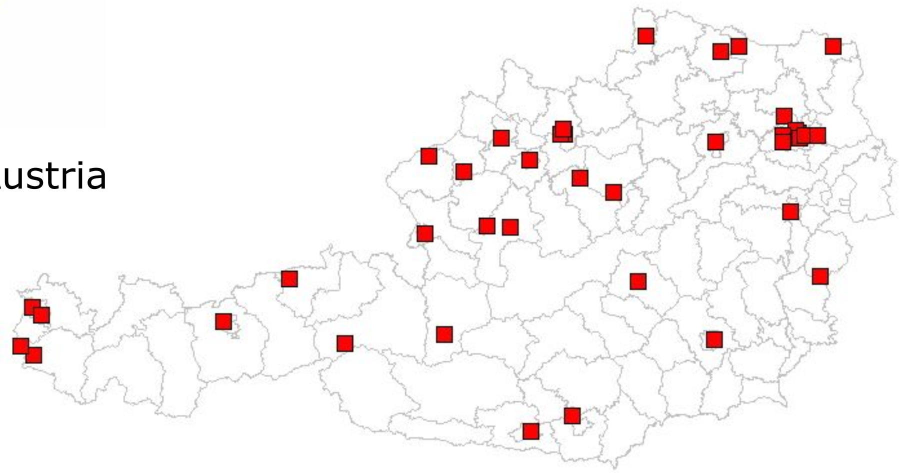


Situation for Cataract Surgery in Austria

Age and gender standardized hospitalisation ratios (SHRs)



Hospitals with department of ophthalmology and optometry



SHR_i... estimate for Relative Risk (RR) to get hospitalized in district i compared to whole Austria

$$SHR_i = \frac{O_i}{E_i}$$

O_i = observed counts in district i

E_i = expected counts in district i assuming the same population structure as in whole Austria

Aim

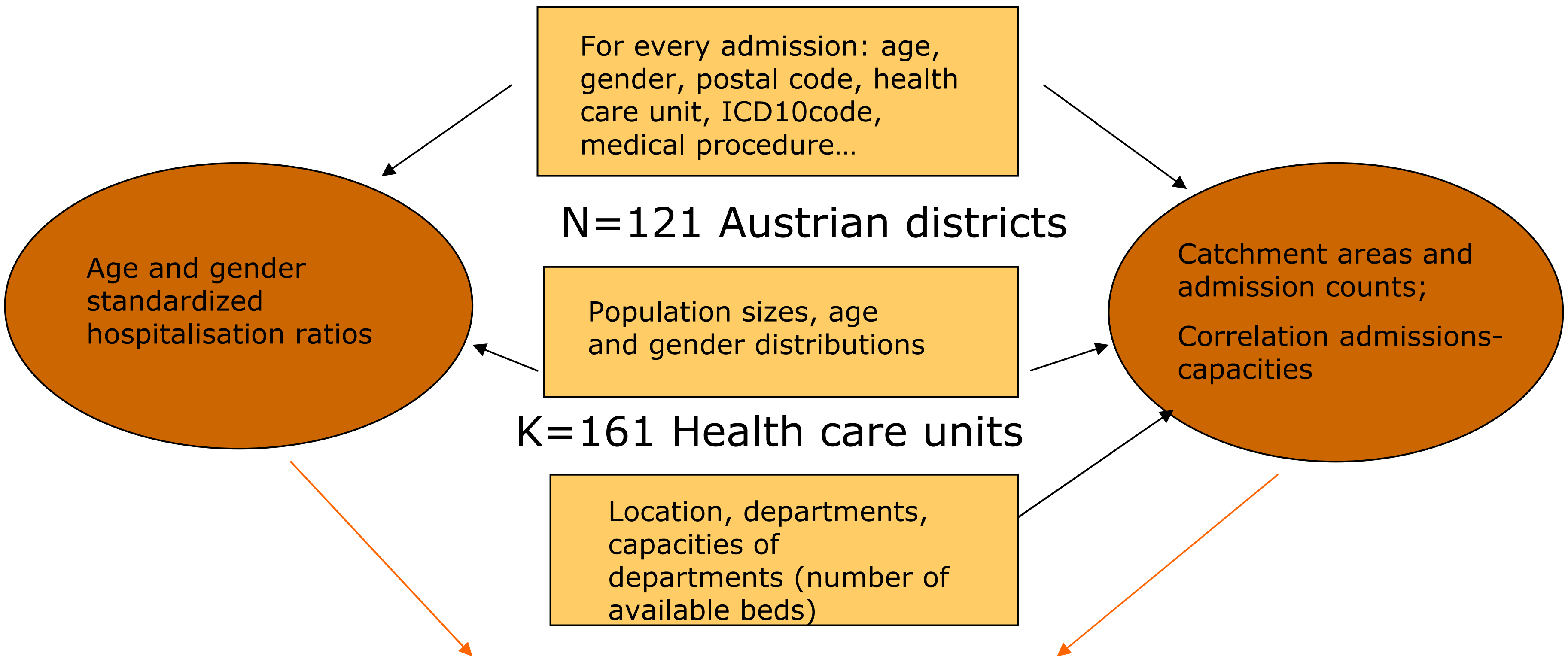
- Is there an area wide health care supply in Austria?
  - What does it depend on?
- How can spatial variability be described?
  - Statistically
  - Graphically
- Are there data available to answer these questions?

Available Data

- Routine hospital data(~2,270,000 hospital admissions / year)
  - Complete
  - Standardized
  - Public and private hospitals
  - Representative for the Austrian health care situation
  - Allow for big retrospective studies
- Health care units
  - Location
  - Departments
  - Capacities (available beds)
- Geographical areas
  - Location
  - Population Counts (census 2001: 8,032,926)

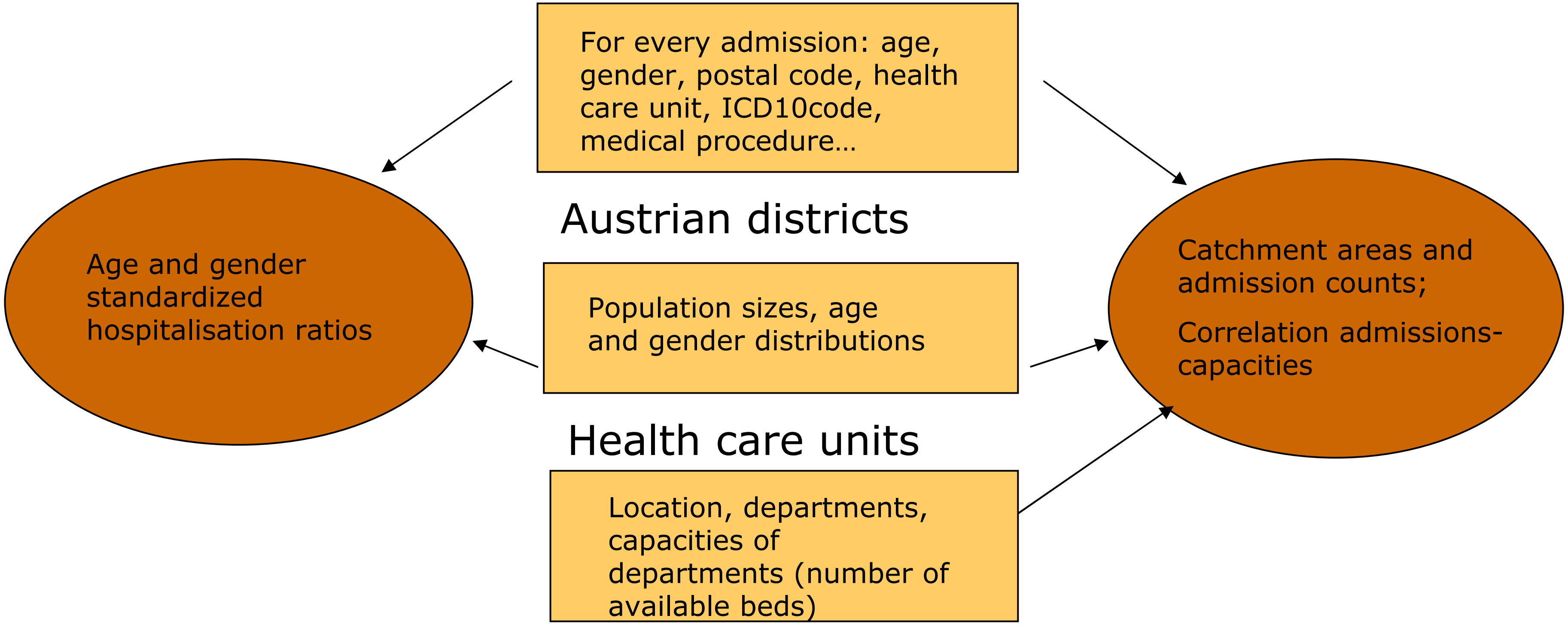
Available Data

Minimum Basic Data Set of the Austrian DRG system



Is there an interaction between SHR_i and available hospitals?
How can it be modelled?

Minimum Basic Data Set of the Austrian DRG system



Aim II – Theoretical Aims

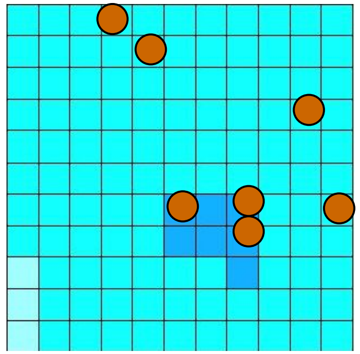
- **Aim**: Explain variation between **districts** by availability of hospitals
- Interaction of (SHRs) districts and hospitals
  - Regarding distances between areas and point sources
  - Capacities of hospitals
- Hospitals interact between each others
- Statistical model that explains this interaction
  - H0: There' s no interaction between hospitals and the SHRs
  - H1: There is an interaction
- Inclusion of all existing information
- Deal with different sample sizes
- Deal with high number of parameters
- Good model fit

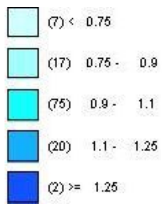
Geographical Considerations

- Alpine regions – flat country - some big cities - many sparsely populated areas
  - Describe Austrian health care situation
- Distance by air distance not adequate distance measure
  - Definition of distance measure district - hospital
  - Mean travelling distance from commune containing a hospital to all communes within a district
- Different district sizes
  - Realistic but general modelling approach

Simulation Data Sets

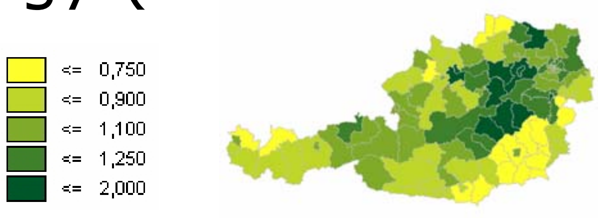
- 11 x 11 grid of squares (total pop. size ~2,000,000)
  - All areas underlying the same risk distribution
  - 2 groups of contiguous areas
  - High heterogeneity
- 7 health care units
  - Fixed position
  - Fixed capacities
  - Fixed distances
- Incidence rate for normal risk IR=0.005
- 3 risk groups
  - Low risk: RR=0.8
  - Normal Risk: RR=1
  - High Risk: RR=1.2





Real Data Sets

- Bypass intervention (3,345)
  - Rare intervention
  - 9 public hospitals with department of heart surgery (K=9)
- Cataract intervention(53,897)
  - Common intervention
  - 39 public and 4 private hospitals with department of ophthalmology (K=43)



Modelling Approach

- Baysian smoothing model
  - Adjust towards overall mean when small sample size
  - Inclusion of prior knowledge
  - Avoid Multiple Testing
- Bayesian Theorem

$$p(\vartheta \mid y) = \frac{p(y \mid \vartheta)\, p(\vartheta)}{p(y)}$$

$p(\vartheta \mid y)$...posterior probability

$p(\vartheta)$... prior probability

$p(y \mid \vartheta)$...likelihood

ϑ... unknown parameter, i.e. RR

Estimating,
not testing!

- Posterior distribution has to be proper
- Calculate the posterior mean

Modelling Approaches I

$O_i \sim pois(\lambda_i E_i)$

O_i...observed counts, E_i...expected counts

λ_i...relative risk in district i

$\log(\lambda_i) = \alpha + h_i + \varphi_i$

$\alpha \sim flat(-\infty, \infty)$...intercept

$h_i \sim normal(0, \tau^2)$...spatial unstructured effect

φ_i...spatial structured effects having a Gausian intrinsic conditional autoregressive prior distribution

$$\varphi_i \mid \varphi_{t \neq i} \sim normal\left(\frac{\sum_t w_{it}\varphi_t}{\sum_t w_{it}}, \frac{\sigma^2}{\sum_t w_{it}}\right)$$

$w_{it} = w_{ti}$...symmetric spatial weights

Model1

Model 1: BYM- model

- No hospital dependence is included
- Term for spatially structured variability
- Term for spatially unstructured variability

Why BYM model?

Parameters estimated on spatial data often correlate with parameters of neighbouring areas. The spatially structured term φ_i for area i regards parameter values $\varphi_{j<>i}$ of the neighbouring areas.

The spatially unstructured term h_i captures random variation in area i, which may depend on parameters not regarded in the model

Conditional Probabilities

- BYM model contains Conditional AutoRegressive (CAR) term, defined by conditional probabilities

$$\varphi_{\mathrm{i}} \mid \varphi_{t \neq i} \sim N\left(\overline{\mu}_i, \overline{\sigma}_i\right)$$

$$\overline{\mu}_i = \frac{\sum_t w_{it}\varphi_t}{\sum_t w_{it}}$$

$$\overline{\sigma}_i = \frac{\sigma^2}{\sum_t w_{it}}$$

σ...overall variance, hyperprior assumed

- This lead to the joint distribution

$$\varphi \sim N(\mu, \Sigma)$$

$$\mu = (0,..,0), i = 1,.., n$$

$$\Sigma_{ii}^{-1} = \frac{\sigma^2}{\sum_t w_{it}}$$

Modelling Approaches II

Model2

$$O_i \sim pois(\lambda_i E_i)$$

$$\log(\lambda_i) = \alpha + h_i(+\varphi_i) + \beta_1 x_{i1} + \beta_2 x_{i2}$$

x_{i1}... – log(distance to the next hospital)

x_{i2}...f(capacity of the next hospital)

BYM+ nearest hospital

- Dependence of the nearest hospital is included
- Distance and capacity are factors

Model3

$$O_{ij} \sim pois(\lambda_{ij} E_{ij})$$

$$\log(\lambda_{ij}) = \alpha - \beta_1 x_{ij} + \beta_2 x_{i2} + \varphi_i + h_i + \eta_j + \psi_{ij}$$

x_{ij}...function of distance/travel time from district i to hospital j

η_j...fixed or random hospital effects

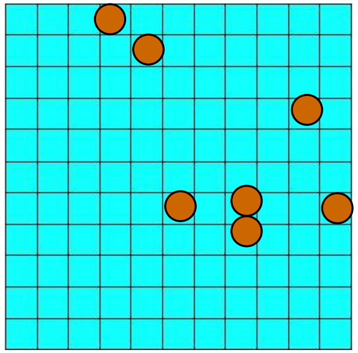
ψ_{ij}...random hospital - geographical area effect

Gravity model

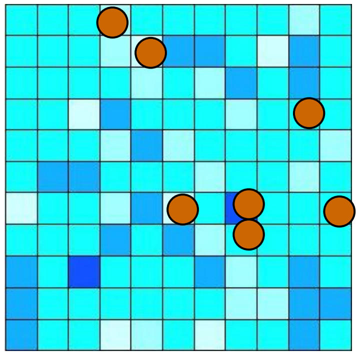
- Patient flows
- To many parameters
- To small sample sizes
- No possibility for graphical description

Same Risk Distribution for all Districts

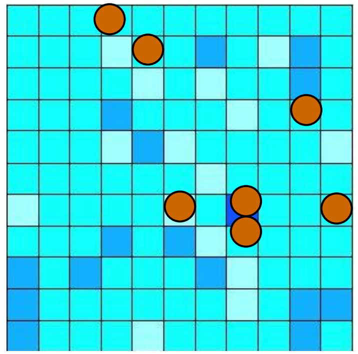
True RR

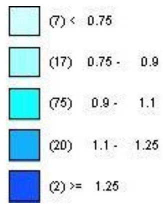


SHR

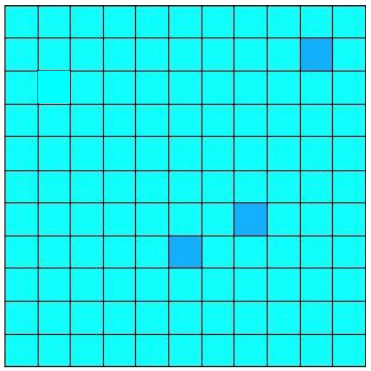


posterior mean of $\alpha + h_i + \varphi_i$





posterior mean of $\alpha + h_i + \beta_1 x_{i1} + \beta_2 x_{i2}$



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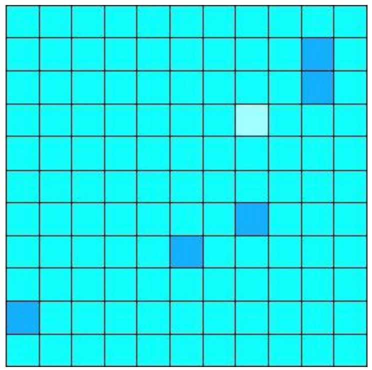
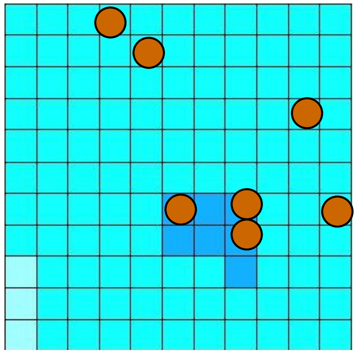
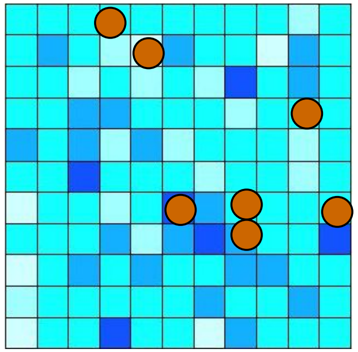


2 Isolated Single Clusters

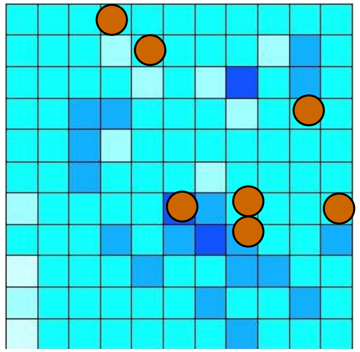
True RR

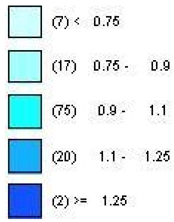


SHR

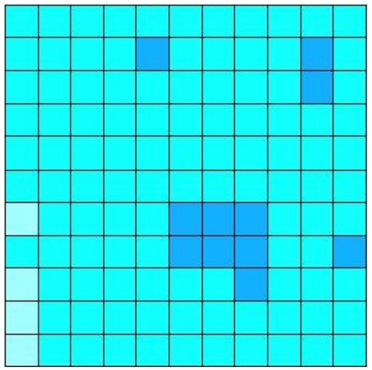


posterior mean of $\alpha + h_i + \varphi_i$





posterior mean of $\alpha + h_i + \beta_1 x_{i1} + \beta_2 x_{i2}$



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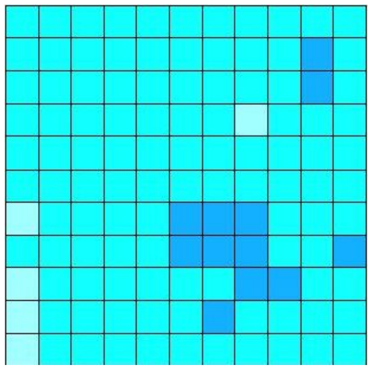
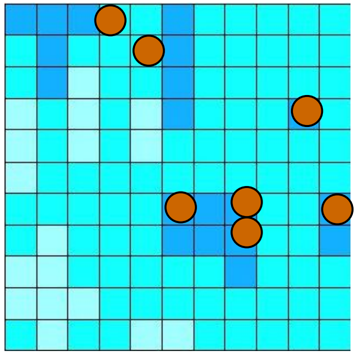
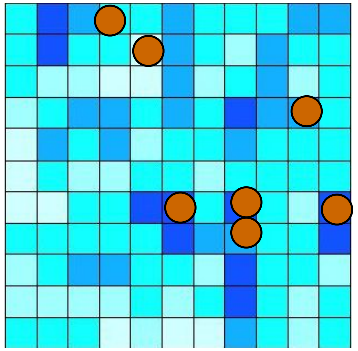


High heterogeneity

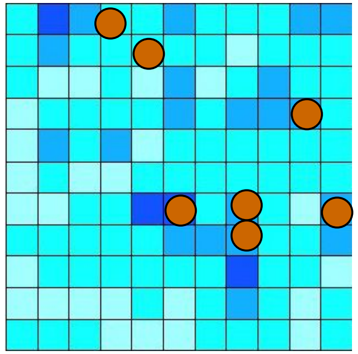
True risk

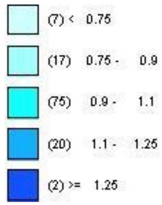


SHR

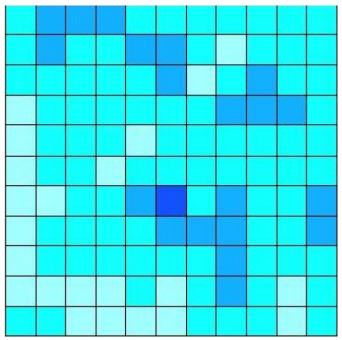


posterior mean of $\alpha + h_i + \varphi_i$





posterior mean of $\alpha + h_i + \beta_1 x_{i1} + \beta_2 x_{i2}$



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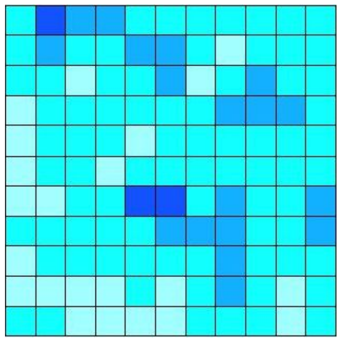
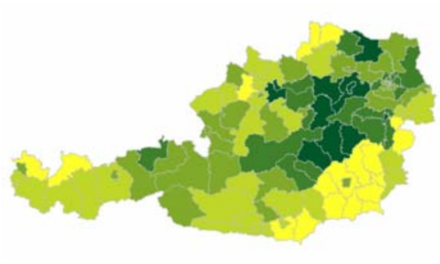


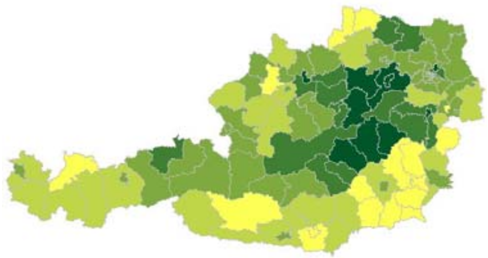
Results Real Data – Cataract Surgery

True risk

SHR



posterior mean of $\alpha + h_i + \varphi_i$



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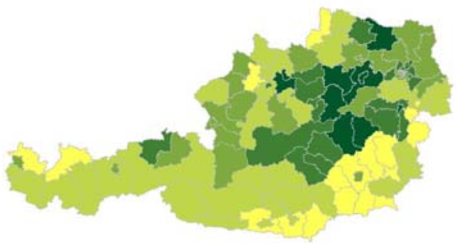

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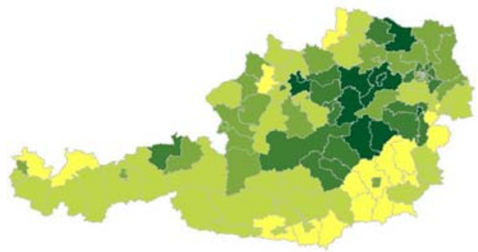
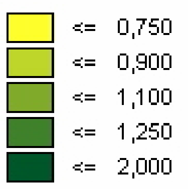
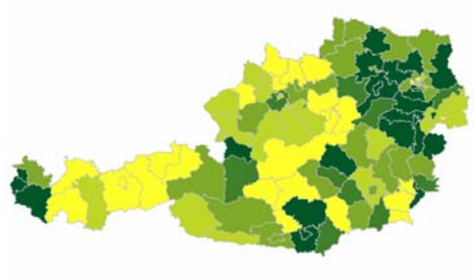




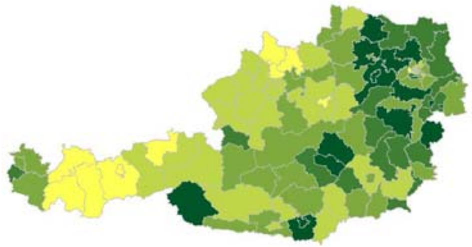
Results Real Data – Bypass Intervention

True risk ?

SHR



posterior mean of $\alpha + h_i + \varphi_i$



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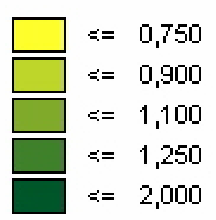

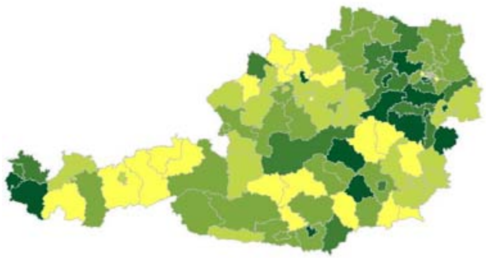

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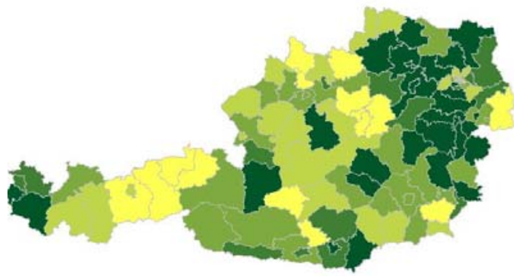


Results - Simulation

DIC (Deviance Information Criterion)

	$\alpha + h_i + \varphi_i$	$\alpha + h_i + \beta_1 x_{i1} + \beta_2 x_{i2}$	$\alpha + h_i + \varphi_i + \beta_1 x_{i1} + \beta_2 x_{i2}$
One risk	924.2	**877.2**	887.6
2 groups of contiguous areas	927.1	**884.1**	891.9
High heterogeneity	930.1	**896.6**	902.8

	$\alpha + h_i + \varphi_i$	$\alpha + h_i + \beta_1 x_{i1} + \beta_2 x_{i2}$	$\alpha + h_i + \varphi_i + \beta_1 x_{i1} + \beta_2 x_{i2}$
Catarct	1143.2	**1142.6**	1149.6
Bypass	**779.4**	797.9	786.6

DIC: Bayesian measure for model comparison, when model applied on the same data

Smallest DIC indicates the best model

DIC regards model fit and model complexity

Interpretation of Results

- Spatial dependence found
- $\alpha + h_i + \beta_1 x_{i1} + \beta_2 x_{i2}$ has less parameters
  - Common intervention - many hospitals
    - Hospitals are important
  - Rare intervention - few hospitals
    - Hospitals are less important
- BYM + nearest hospital
  - too much
- Inclusion of more than one hospital as fixed effect
  - Wrong approach – no hospital interaction

Theoretical Relevance

- BYM
  - Spatial **structured** term
  - **Unstructured** heterogeneity
- Hospital effect
  - important
- Gravity model
  - **All** interactions are regarded
  - **Patient flows** between hospitals and districts

Problems and Questions

- BYM model does not explain all spatial variation
- Effect of the nearest hospital not enough
  - All hospitals as fixed effects - no between hospital interaction
- Gravity model has to many parameters
- How can hospital district interaction be modelled?
  - How many parameters?
  - Graphics?
- Regard all hospitals in all districts?
  - Definition of a threshold (admission proportion=5%)
- Literature proposes fixed effect term
  - averages across all hospitals in each district

New Approach - Idea

- **Aim**: Explain supply within districts by availability of hospitals
- **Idea**: Use advantages of different models
- Extend BYM by term of structured district-hospital effect
  - Spatially structured effects necessary?
- Regard
  - Distances d_{ik} (district i and hospital k)
  - Capacities c_k
  - Population size s_i

→ patient flows

- Between hospital interaction
- Not calculate mean over hospitals

Requirements

- Readdress district hospital interaction into problem of patient flows between districts
  - Describe by N x N matrix Δ
- Δ=SDC: availability matrix
- Δ not symmetric
- D= $(f(dist_{ij}))_{ij}$: N x N distance matrix
  - D not symmetric
  - Functions $f(dist_{ij})$ of distances from district i to hospital(s) in district j
- C: N x N diagonal capacity matrix
  - Functions $h(c_j)$ of capacities of the hospitals in district j
- S: N x N diagonal population matrix
  - Functions $g(s_i)$ of population size of district i

Mathematical Formulation I

- Model term for structured area – hospital interaction η
  $\log(\lambda_i) = \alpha + h_i + \eta_i$
- Let's assume $\eta_i \mid \eta_{j \neq i} \sim \mathrm{N}(\mu_i, \sigma_i)$
  - District i supplied well
    - Small variation σ_i
  - District i supplied not so well
    - High variation σ_i
- How does joint distribution of prior η look like?
  - $\eta \sim N(\mu, \Sigma)$
  - How is Σ defined?
  - Proper posterior?
  - Is it possible to use a prior like the CAR prior

Comparison with CAR model

Δ... Availability matrix new model	Δ ...Distance matrix CAR model
Δ not symmetric $\Sigma^{-1} = M(I - R^{-1}B) = \sigma(R - B) \neq \sigma(C - B)$ $M = \sigma R$ σ... overall Variance R... Diagonalmatrix with A_{ii} rowsums of Δ C... Diagonalmatrix with B_{ii} columsums of Δ	Δ symmetric: $\Sigma^{-1} = M(I - R^{-1}\Delta) = \sigma^{-1}(R - \Delta) = \sigma^{-1}(C - \Delta)$ $M = \sigma^{-1}R$ σ... overall Variance R... Diagonalmatrix with R_{ii} rowsums of Δ C... Diagonalmatrix with C_{ii} columsums of Δ
Σ^{-1} not symmetric, diagonal elements **should contain** variances of conditional probabilities	Σ^{-1} symmetric, diagonal elements **contain** variances of conditional probabilities
Some diagonal elements $\Delta_{ii} \neq 0$	Diagonal elements $\Delta_{ii} = 0$
All elements $\Delta_{ij} \geq 0$	All elements $\Delta_{ij} \geq 0$

Statistical Background

- Definition of $\Lambda = \Sigma^{-1}$

$$\Lambda = \sigma^{-1} A (I - A^{-1} \Delta B^{-1}) B$$

I...Identity matrix

A...diagonal matrix with $A_{ii} = \sqrt{\sum_{j=1}^{n} \Delta_{ij}}$ Row sums of Δ

B...diagonal matrix with $B_{ii} = \sqrt{\sum_{j=1}^{n} \Delta_{ji}}$ Column sums of Δ

σ...common variation

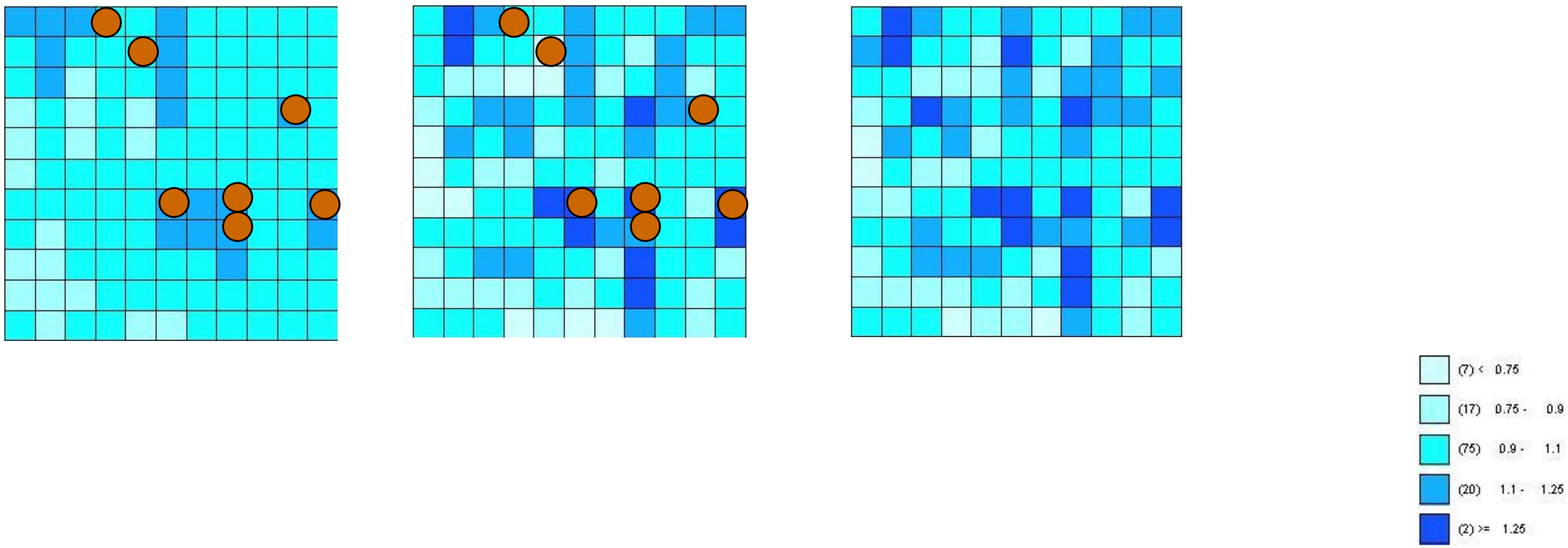
- Properties of $\Lambda = \Sigma^{-1}$
  - Symmetry
    - Can be symmetrised
  - Rank
    - Strictly diagonal dominant ⇒ full rank
  - Positive definite / semi definite
    - Strictly diagonal dominant ⇒ positive definite
    - Weak diagonal dominant ⇒ positive semi definite

Mathematical Formulation – 2 Solutions

Symmetrise availability matrix Δ: $W=1/2(\Delta+\Delta^T)$	Symmetrise inverse variance covariance Matrix $\Lambda=\Sigma^{-1}$: $\Sigma_{new}^{-1}=1/2(\Lambda+\Lambda^T)$
Σ_{new}^{-1} weakly diagonal dominant	Σ_{new}^{-1} strictly diagonal dominant and $\Leftrightarrow \quad 2A_iB_i > \sum_{j=1}^{N}(A_i^2+B_i^2)$ Especially A_{ii}, B_{ii} >1
$\Delta_{ii}\neq 0 \quad \forall j\neq i \quad \Rightarrow$ Set $\Delta_{ii}=0$ and $\eta_i \| \eta_i \sim N(\mu_i,\sigma_i^2)$ with	$\Delta_{ii}\neq 0 \quad \forall j\neq i \quad \Rightarrow$ Set $\Delta_{ii}=0$ and $\eta_i \| \eta_i \sim N(\mu_i,\sigma^2_i)$ with
$\mu_i = \alpha\Delta_{ii} + \frac{1}{\frac{1}{2}(A_{ii}^2+B_{ii}^2)}\sum_{j\neq i}\Delta_{ij}\mu_j$ $\sigma_i = \frac{2\sigma}{A_i^2+B_i^2}$	$\mu_i = \alpha\Delta_{ii} + \frac{1}{A_{ii}B_{ii}}\sum_{j\neq i}\Delta_{ij}\mu_j$ $\sigma_i = \frac{\sigma}{A_{ii}B_{ii}}$

Results- Simulation

True risk SHR Posterior by new model



Conclusion

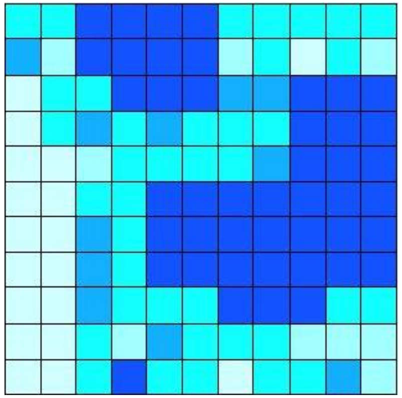
- New model regards between hospital interactions as well as hospital district flows
- Posterior distribution is proper
- Computational amount does not depend on the number of hospitals
- Not more parameters than in BYM model
- Variation depends on availability of hospitals

Questions and Discussion

- More than one hospital in a district → average across these hospitals
- Symmetric solution for a non symmetric problem
- Within district patient flows not regarded in the variance
- Identifiability
  - How to describe graphically the term of hospital dependence?
- What are the best priors?
- Is there a better model?
- Should other parameters be considered?
- Is between hospital interaction regarded in the right way?

Further Work

- Simulations on different data sets
  - Map of Austria with real pop. size
  - Ideal 11x11 grid
- Investigations on the best hyperpriors
- Critical discussion on chosen functions and thresholds
- Comparison with existing methods
- Investigations on alternative methods



Thank you for your Attention!

Non Symmetric Approach

- η of length N
- N areas – K hospitals
  - Interaction not by symmetric NxN matrix
  - η of size (NxK) – too many parameters
- Introduce parameter vector ψ of length K with

$$\eta_i \mid \psi_{j=1..K} \sim N(\mu_{\eta i}, \sigma_{\eta i})$$

$$\psi_j \mid \eta_{i=1..N} \sim N(\mu_{\psi j}, \sigma_{\psi j})$$

- N+K parameters

Statistical Model

$$O_i \sim pois(\lambda_i E_i)$$

$$\log(\lambda_i) = \alpha + h_i + \varphi_i + \eta_i$$

h_i...$normal(0, \tau^2)$

φ_{i}...spatial structured effects having a Gausian intrinsic conditional autoregressive prior distribution

$$\varphi_{\mathrm{i}} \mid \varphi_{t \neq i} \sim N\left(\frac{\sum_t w_{it}\varphi_t}{\sum_t w_{it}}, \frac{\sigma^2}{\sum_t w_{it}}\right)$$

w_i...spatial weights

η_{i}...area – hospital effect

$$\eta_{\mathrm{i}} \mid \psi_{j=1..K} \sim N(\mu_{\eta i}, \sigma_{\eta i})$$

$$\psi_j \mid \eta_{i=1..N} \sim N(\mu_{\psi j}, \sigma_{\psi j})$$

Requirements and Definitions

- Proper posterior for λ
- η|φ and φ|η unique

$$\mu_{\eta i} = (\Delta C)_i \psi \qquad i = 1,..,N$$

$$\mu_{\psi j} = (S\Delta)_j^T \eta \qquad j = 1,..,K$$

$$\sigma_{\eta i} = \frac{\sigma_\eta}{a_{\eta i}}$$

$$\sigma_{\psi i} = \frac{\sigma_\psi}{a_{\psi i}}$$

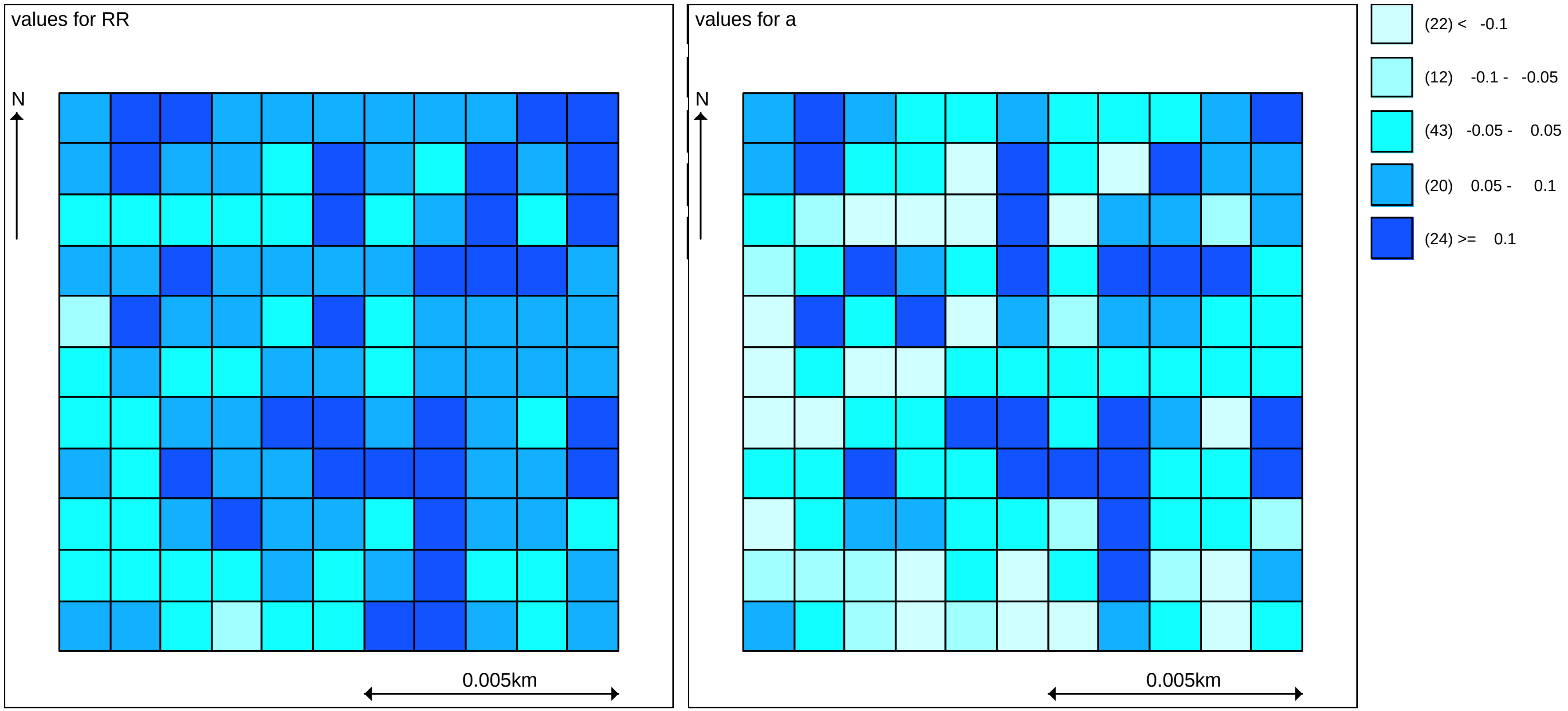
σ_η, σ_ψ hyperpriors

Mathematical Problems

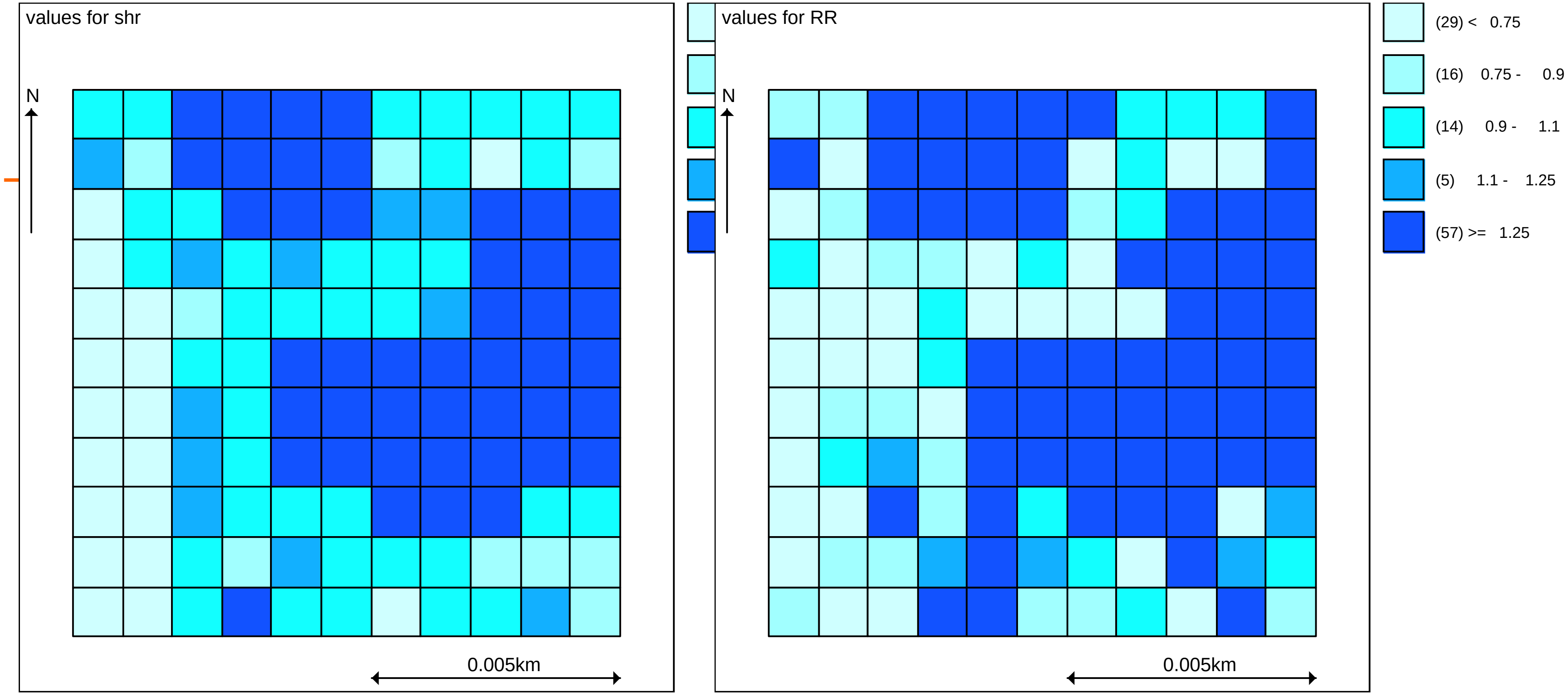
- Mean η is not around 0
  - Can it be standardized towards 0?
- Can ΔC and $S\Delta$ have any structure
  - Do we need conditions on these matrices?
- Are normal distributions the right priors for the conditional probabilities?

Implementation

Results- Simulation non symmetric model

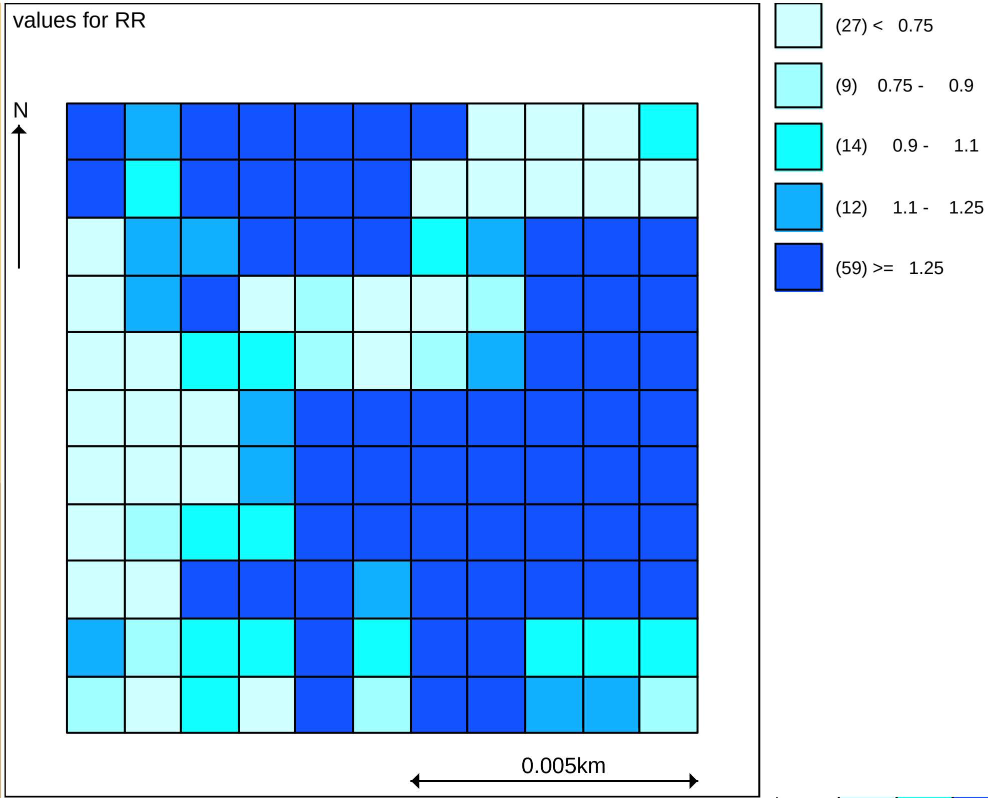


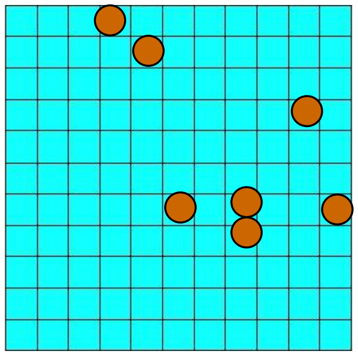
Model Comparison

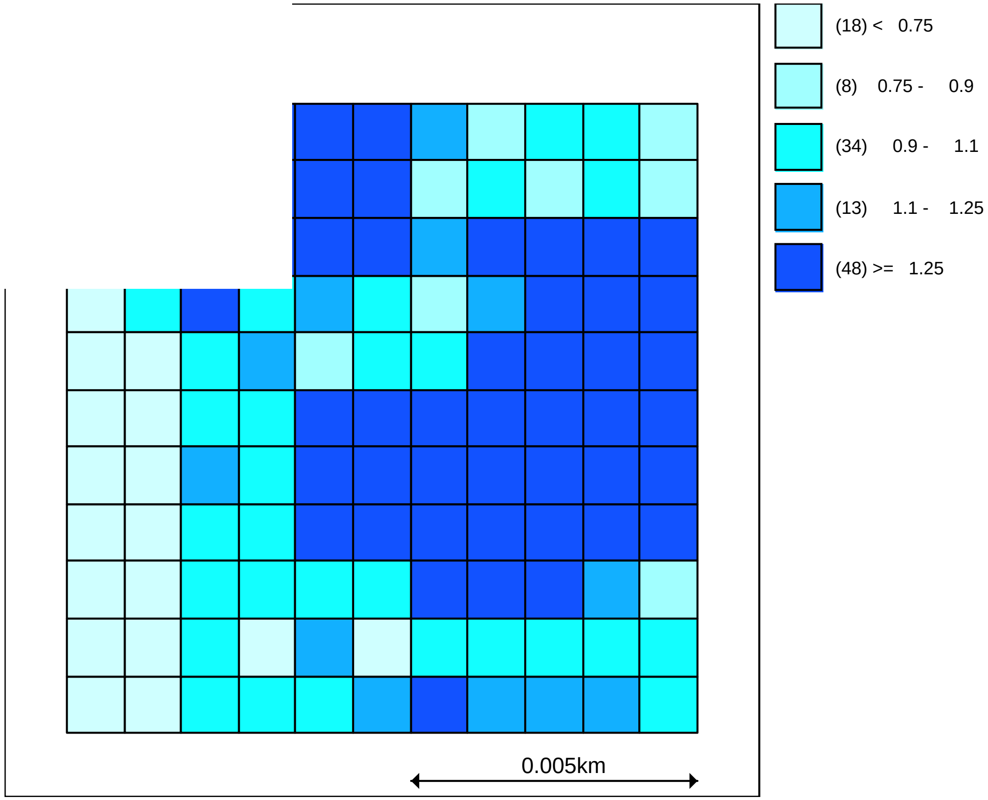


Dbar = post.mean of -2logL; Dhat = -2LogL at post.mean of stochastic nodes

	Dbar	**Dhat**	**pD**	**DIC**
O	958.146	839.757	118.389	1076.540
total	958.146	839.757	118.389	1076.540


values for RR
N
0.005km
(27) < 0.75
(9) 0.75 - 0.9
(14) 0.9 - 1.1
(12) 1.1 - 1.25
(59) >= 1.25




0.005km
(18) < 0.75
(8) 0.75 - 0.9
(34) 0.9 - 1.1
(13) 1.1 - 1.25
(48) >= 1.25