NEUROPSYCHOLOGICAL REHABILITATION, 2004, *14* (3), 303–328

Adding colour to multiplication: Rehabilitation of arithmetic fact retrieval in a case of traumatic brain injury

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This study describes ME, a patient in the chronic stage after a traumatic brain injury. During an extensive training programme ME tried to regain automaticity in the retrieval of simple multiplication facts. He succeeded in substantially decreasing response latencies in multiplication, reducing the handicap at his job. This improvement generalised to a non-trained operand order, to non-trained problems, and to a non-trained output modality. Moreover, these effects were maintained over at least four months. Interestingly, however, ME's training effects were operation specific: No significant improvement occurred in addition, subtraction, or division. As coloured presentation of multiplication problems proved to be a valuable cue in facilitating the patient's performance, this might turn out to be a useful tool in the rehabilitation of fact retrieval in general.

INTRODUCTION

Arithmetic facts (i.e., basic arithmetic operations involving digits from 2 to 9) form an essential part of our cultural equipment. A lack of this knowledge thus results in a severe handicap. However, despite the fact that arithmetic facts

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We are indebted to the continuing participation of ME.

This research was supported by RTN European Community grant HPRN-CT-2000-00076 and the Austrian Federal Ministry for Education, Science, and Culture.

http://www.tandf.co.uk/journals/pp/09602011.html DOI:10.1080/09602010343000246

are lost in a substantial number of brain damaged patients (e.g., Delazer, Girelli, Semenza, & Denes, 1999; Grafman, Passafiume, Faglioni, & Boller, 1982; Hécaen, Angerlergues, & Houllier, 1961), relatively few studies address the rehabilitation of such a loss (Girelli, Batha, & Delazer, 2002; Girelli, Delazer, Semenza, & Denes, 1996; Hittmair-Delazer, Semenza, & Denes, 1994; Kashiwagi, Kashiwagi, & Hasegawa, 1987; Miceli & Capasso, 1991; Whetstone, 1998; and, more anecdotally, McCloskey, Aliminosa, & Sokol, 1991).

Traditionally, the rehabilitation of facts is obtained by drill, i.e., by repeated exposure to the problems (e.g., Girelli et al., 1996; Hittmair-Delazer et al., 1994; Kashiwagi et al., 1987; Miceli, & Capasso, 1991; Whetstone, 1998). In most of the cases this drill consisted of repeated production of the solution with immediate feedback provided. However, patient MC of Whetstone (1998) relearned problems together with their solutions presented in different modalities.

The rationale for the repeated exposure approach is based on the concept that facts are stored in declarative memory as associations between the terms of the problem and the answers (Ashcraft, 1992; Campbell, 1995; McCloskey & Lindemann, 1992; Rickard & Bourne, 1996; Siegler, 1988). Hence, repeated exposure is thought to re-establish such connections by strengthening these associations. Indeed, it has been shown that after extensive repetition with feedback, drill led to a change in the nature of errors which became more plausible or closer to the target (Girelli et al., 1996), to a significantly decreased error rate (Miceli & Capasso, 1991; Girelli et al., 1996), and even to performance near or at ceiling (Hittmair-Delazer et al., 1994; Whetstone, 1998). However, errorless performance does not necessarily prove complete restitution of function. In the case of MC (Whetstone, 1998) response latencies between 3568 ms (spoken verbal input, match condition) and 6060 ms (written verbal input, non-match condition) clearly indicate effortful retrieval or the application of back-up strategies. Thus, recording and analysing response latencies might be a valuable tool to describe a patient's profile in much more detail. Nevertheless, this was only done in some systematic fashion by Hittmair-Delazer et al. (1994) and Whetstone (1998).

Generalisation to non-trained problems differed to some extent in the studies mentioned above. Hittmair-Delazer et al. (1994) found training of their patient BE to result in specific reaction time (RT) effects, not fully generalising to complements of trained facts (e.g., 3×4 to 4×3). Nevertheless, BE was able to apply the relearned tables in complex written calculation as well as in division problems.

Girelli et al. (1996) reported main effects of treatment for the respective training sets of patients TL and ZA. In addition to this, however, an improvement was observed for both patients, also for the non-trained set of multiplication problems (between T1 and T2). Furthermore, the effects of multiplication training enabled both patients to solve most of the simple division problems.

Analysing data of his patient MC, Whetstone (1998) found no effect of training presentation format on error rates, probably due to a ceiling effect. However, reaction time data revealed a match effect: problems were answered faster in the trained modality (e.g., written) than in the untrained modality (e.g., spoken). Interestingly, MC showed a complete generalisation to the non-studied operand order, although problems were only presented in one operand order during training. No effects on division are reported.

Drill is not the only way to improve the knowledge of arithmetic facts. Recently, Girelli et al. (2002) reported a successful rehabilitation of simple arithmetic by teaching their patient FS the application of strategies. The rationale of such an approach is based on evidence for different pathways subserving the solution of simple arithmetic facts (e.g., Baroody, 1994; Siegler, 1988). In addition to retrieval from memory (and preceding it in development) algorithms and strategies which are based on procedural and conceptual knowledge can be used to solve such problems. Although not re-establishing retrieval, a conceptual approach is potentially able to reach the goal with much less training and with good generalisation to non-trained problems.

In general, the choice of an appropriate rehabilitation method might be guided by the patient's profile (e.g., Lochy, Domahs, & Delazer, in press): If a patient has only very limited abilities to store new material in declarative memory, (re-)teaching him conceptual knowledge or the use of strategies will be the method of choice. Advantages of this method are a flexible knowledge, leading to good generalisation and the possibility to create a rich conceptual network. However, the patient has to rely on some fronto-executive abilities including working memory. If a patient already uses strategies, and faster retrieval is the aim, the restoration of fact retrieval by drill might be the preferred option. The impairment of the latter kind of patients may *only* manifest itself as increased response latencies (Warrington, 1982), again pointing to the value of documenting reaction times. Given such conditions, Hittmair-Delazer et al. (1984) successfully adopted a training under time pressure for their patient BE.

Not all simple multiplication problems are considered as arithmetic facts. McCloskey and coworkers (McCloskey, Caramazza, & Basili, 1985; McCloskey et al., 1991) distinguished between three subsets of simple multiplication problems: 0 problems (all problems involving 0 as operand), 1 problems (all problems involving 1 as operand) and 2–9 problems (2×2 through 9×9). While the first two subsets (involving 0 and 1) are thought to be answered by a stored rule, only the problems of the last subset are thought to be stored individually and to be retrieved from memory. Problems involving 0 ($0 \times n$; $n \times 0$) or 1 ($1 \times n$; $n \times 1$) typically show consistent error patterns (e.g., $n \times 0 = n$; $n \times 1 = 1$), 0 problems seeming to be more error prone than 1 problems

(McCloskey et al., 1991).[1] While in the present study rule-based problems were not included in the set of trained items, they were part of the baseline and testing sessions.

In the rehabilitation of declarative memory, attempts have been made to support explicit retrieval not only by drill, leading to a restoration of function, but also by additional involvement of implicit memory functions. One prominent such account is the method of vanishing cues. This method has been successfully applied in the rehabilitation of memory impaired patients. Originally, it was adopted for the acquisition of computer-related vocabulary by amnesic patients (Glisky & Schacter, 1989; Glisky, Schacter, & Tulving, 1986). Subsequently, it was also applied to other tasks such as, for example, learning people's names (Diesfeldt & Smits, 1991; Thöne & Glisky, 1995). The method involves the systematic reduction of cues, in most cases letter fragments of to-be-learned words across trials. As Glisky et al. (1986) pointed out, learning with this method is slow and strongly dependent on cues, but eventually successful, so that even severely amnesic patients were able to learn and retain computer-related knowledge. While the technique originally proposed by Glisky et al. (1986) was though to rely on implicit memory, other studies emphasise the contribution of explicit memory (e.g., Hunkin & Parkin, 1995).

In the present study, colours are used as cues to support explicit retrieval of arithmetic facts from memory. Colours are frequently used in rehabilitation to draw the patient's attention to a certain stimulus. Deloche, Seron, and Ferrand (1989), for instance, described the use of colours to illustrate different powers of 10 in the treatment of number transcoding. Interestingly, numbers have been found to be associated to colour in mental representations of some subjects (Galton, 1880a, 1880b; Seron et al., 1992). Even mental arithmetic can be associated to colour experience in healthy people with synesthesia (Dixon, Smilek, Cudahy, & Merikle, 2000; Smilek, Dixon, Cudahy, & Merikle, 2002). In their synesthetic subject C, these authors demonstrated the automatic induction of photisms (i.e., colour experiences) associated to a digit by presenting an arithmetic problem for which this digit is the solution. C was substantially slower to name colour patches incongruent with photisms associated to the solution than to name colour patches congruent to the solution (e.g., presented problem: 5 + 2, photism associated to 7: yellow, congruent colour naming: yellow, incongruent colour naming: red).

[1] It has also been shown that rules for addition ($n + 0$; $0 + n$) may de disrupted selectively. Pesenti, Depoorter, and Seron (2000) described a patient who answered all rule-based problems well with the exception of $0 + n$ problems. Thus, the non-commutability of rules, earlier described for multiplication, also applies to addition.

In the following study the retraining of simple multiplication facts will be described in a patient who showed severely increased efforts to retrieve these facts despite low error rates, due to the application of time-consuming calculation strategies. The training aimed at regaining fluency of retrieval, indicated by a decrease of response latencies. Accordingly, it consisted of repeated exposure to the problems (e.g., Lochy et al., in press). Colour was provided as visual cue, vanishing in two steps. This might on the one hand reduce the amount of errors, leading to peaked associations between problems and answers (Siegler, 1988). On the other hand, coloured presentation may serve to link the solution of arithmetic problems to a different kind of information (i.e., colour), thus directly facilitating retrieval. The outcome will be evaluated for the trained problems and possible generalisation effects will be examined. Importantly, it will be investigated how coloured cues vanishing in two steps are able to support retrieval.

CASE DESCRIPTION

We report the case of a 38-year-old right-handed man, ME. He was working as a car mechanic when he had a traumatic brain injury about three years before our examination. This accident caused a left frontal contusion. At the time of our examinations he had recovered substantially, working on a part-time basis in the purchasing department of his former employer. No former problems in number processing or any other cognitive domain were manifest before the accident. However, at the onset of our examinations he was described to be severely handicapped at his new job, due to slow and effortful calculation.

NEUROPSYCHOLOGICAL AND LANGUAGE EXAMINATIONS

The patient showed slightly compromised verbal working memory and learning abilities whereas his figural memory was found to be intact. Computerised tests revealed a reduced alertness and divided attention. This manifested in increased and fluctuating latencies. Alertness did not improve from cued presentation. ME presented with impaired cognitive flexibility and divergent thinking. He showed no signs of apraxia or visual-constructive disorder. An overview of his neuropsychological test results is given in Table 1.

The patient's language performance was characterised by slightly impaired naming and word finding in confrontation naming and fluency tasks. He showed a slow, often syllabic reading and some surface dysgraphic writing errors. An overview about his result in language tests is given in Table 2.

A comprehensive number processing battery (Delazer, Girelli, Graná, & Domahs, in press) revealed a few problems in complex tasks like complex mental or written calculation and the application of arithmetic principles (see

TABLE 1
Neuropsychological background tests in August 2001

Domain/test	*Raw score*	*Max. possible score or cut off*
Memory		
Wechsler Memory Scale		
Digit span forward	4*	7
Digit span backward	6	7
California Verbal Learning Test		
Learning trials 1–5	4, 7, 8, 9, 10*	16
Short delay free recall	6**	16
Short delay cued recall	7**	6
Long delay free recall	5**	16
Confabulations	4*	
Recognition	14	16
Perseveration	3*	
Rey Osterrieth Complex Figure		
Long delay figural free recall	25	36
Rivermead Behavioural Memory Test		
Story short delay recall	1**	21
Story long delay free recall	1.5**	21
Visuoperception and construction		
Rey Osterrieth Complex Figure		
Copy	34	36
Hooper Visual Organization Test		
Visual organization	22.5 (corrected)	30
Executive functions		
Trail Making Test		
Test A	36.5 s	
Test B	99.9 s**	
Five-Points Test		
Overall number of figures	42	
Perseveration (in %)	14.3	15 (cut)
OMO		
Set shifting errors	1	3 (cut)
HAWIE-R		
Picture sequencing	21*	
Conceptualization	15**	
Praxis		
De Renzi Apraxia Test	70	72
Attention		
TAP Test of Attention performance		
Alertness med RT	1197 ms**	
Divided attention med RT	1273 ms**	

* Score between 11th and 25th percentile or moderately impaired performance. ** Score of 10th percentile and below or severely impaired performance. Cut off is indicated by (cut)

TABLE 2
Neurolinguistic background tests in August 2001

Test	*Raw score*	*Max. possible score*
Aachener Aphasie Test		
Token Test (errors)	2	50
Comprehension	105	120
Repetition	146	150
Naming	114	120
Written Language	76*	90
IBT Naming	40*	50
Verbal Fluency		
Animals (per 20 items)	150 s*	
S-words (per 20 items)	143 s*	
Alternating (per 20 items)	ceased	

* Score between 11th and 25th percentile or moderately impaired performance.

Table 3). On the other hand, basic number processing abilities like transcoding from one format into another or simple calculation appeared to be largely intact. However, the patient showed extreme response latencies of up to 90 s for simple multiplication facts, which were partly due to the (overt) use of strategies instead of simple retrieval from memory.[2] In a subsequent test, ME was asked to guess the solution of 64 simple multiplication problems within a maximum of 10 sec each. With this time limit, accuracy dropped considerably. He scored only 39/64 (60.9%) correct. For 31/39 correctly solved problems it took him 3–10 seconds to answer. Only 8/39 of these problems were answered in less than three seconds, possibly reflecting fact retrieval. Interestingly, retrieval in this sense was limited to problems of the 2 times table and to tie problems. Fast answers to large tie problems (e.g., 6 × 6 and 8 × 8) in the context of slow answers to comparably large non-tie problems (e.g., 4 × 9 and 9 × 7) can be interpreted as further support for true retrieval as ME obviously could not apply fast strategies for large problems.

The patient used strategies like changing the operand order (e.g., 7 × 3 to 3 × 7), decomposing problems (e.g., 7 × 8 = 5 × 8 + 2 × 8; 6 × 9 = 6 × 10 – 6), repeated addition (6 × 3 = 6 + 6 + 6) or counting on from a known result in the table (e.g., 6 × 8 = 40 (5 × 8) –41 –42 –43 –44 –45 –46 –47 –48). ME was handicapped by his slow and effortful procedures for solving simple multiplication facts, because it prevented him for efficiently doing his job in the purchasing department.

[2] In fact, retrieval in healthy young subjects occurs within one second or less (e.g., Campbell, 1987b; McCloskey et al., 1991; McCloskey, 1992).

TABLE 3
Number processing battery (Delazer et al., in press), August 2001

Domain/test	*Score*	*Max. possible score*
Counting	8	9
Parity judgements	10	10
Number comparison		
Arabic numbers	20	20
Written number words	8*	10
Spoken number words	10	10
Reading		
Arabic numbers	17	18
Number words	10	10
Writing		
Arabic numbers to dictation	18	18
Arabic numbers from written number words	9*	10
Number scale matching		
Arabic numbers	5	6
Written number words	6	6
Token transcoding		
Tokens → Arabic numbers	10	10
Arabic numbers → tokens	10	10
Simple arithmetic facts		
Addition	110	110
Subtraction	117	120
Multiplication	98	100
Division	112	120
Complex mental calculation		
Addition	5	5
Subtraction	2*	5
Multiplication	5	5
Division	3	5
Complex written calculation		
Addition	0*	3
Subtraction	1*	3
Multiplication	2*	3
Text problems	10	12
Arithmetic principles		
Addition	12*	15
Multiplication	9*	15

* Indicates score between 11th and 25th percentile or moderately impaired performance.
Simple arithmetic facts were not tested using the battery of Delazer et al. (in press). The reason was to include more problems (e.g., problems in both operand orders instead of only one).

REHABILITATION

The target of the intervention was to improve the patient's everyday situation, especially his ability to cope with the demands of his job. Given that he successfully used strategies to solve simple multiplication problems, the main aim was to induce a shift from these time-consuming strategies to a restoration of function, i.e., fast and effortless fact retrieval. Importantly, such a shift would be evidenced mainly by response latencies rather than by error rates. Extensive exposure should strengthen the associations between the problems and their solutions and thus allow more automatised retrieval. In addition to similar approaches in the literature (Girelli et al., 1996; Hittmair-Delazer et al., 1994; Kashiwagi et al., 1987; Whetstone, 1998) a visual cue was provided to the patient in the initial sessions to reduce the risk of erroneous associations establishing.

Method

Baseline and test procedures

Verbal production. The patient had to answer orally simple arithmetic problems of all four operations (addition, subtraction, multiplication, and division) which were presented randomly on a computer screen blocked by operation. Problems included all possible combinations of operands from 0 to 10 (i.e., including rule-based problems). For equivalent problems both complementary operand orders were presented (e.g., 3×4 and 4×3; $3 + 4$ and $4 + 3$). Ties (e.g., 3×3) were only presented once. Response latencies were registered by a voice key and accuracy was documented by an examiner. No feedback on accuracy was provided to the patient during baseline and test examinations.

Keyboard production. Multiplication problems were presented to the patient on a computer screen and he had to enter the solution on a computer keyboard. The same problems were presented three times to the patient, blocked by type of presentation: During the first block, problems were presented in colour as specified below, during the second block, problems were presented in black, and during the third block, problems were presented in colour again. Within each block, problems were presented in randomised order. The design of two blocks of coloured presentation before and after black presentation was chosen to minimise the impact of possible learning effects within one session. Response latencies for coloured presentation will be reported as mean RTs of both blocks.

Colour digit associations. At three different occasions ME was asked whether he could spontaneously associate colours with single digits.

Training

Training took place in two phases. Based on data of the verbal baseline examination, two subsets of 14 multiplication problems each (set A and B) were chosen to be matched as closely as possible for the patient's response latencies and accuracy. The sets did not differ significantly in terms of response latencies or accuracy. During phase 1, subset A was trained and subset B served as control, during phase 2 it was the reverse. Both subsets consisted of arithmetic facts only, excluding rule-based problems (i.e., factors 0, 1, and 10). Operand order was restricted to "large times small" type (e.g., 7 × 3 instead of 3 × 7). No tie problems were presented for training (e.g., 7 × 7). Problems were presented in Arabic format on a computer screen and were to be answered on the keyboard. Feedback was provided every time a wrong digit was pressed. In these cases, the wrong digit was not displayed. The correct solution was given on the screen and the trial was repeated.

Training usually took place three times per week. Phase 1 consisted of 20 sessions. During each session, the set of 14 problems was presented five times (i.e., in five blocks). Problems were randomised within each block of presentation. Phase 2 started approximately one month after the end of phase 1. It consisted of 25 sessions, because the patient became ill for two weeks after session 18 (see Figure 1).

During the first 10 sessions of both phases, problems were presented in nine different colours.[3] Each colour was associated to the second digit (i.e., the unit) of the solution. In this way, problems 4 × 3, 6 × 2, 8 × 4, 7 × 6, and 9 × 8 were all presented in yellow, because the second digit of their solutions (12, 12, 32, 42, and 72, respectively) is 2 and 2 was associated with yellow. The principle of this mapping was explained to ME before the beginning of the training, but after baseline testing. Furthermore, during the first five sessions of both phases a legend was provided on the edge of the screen indicating the digit each colour was associated with. From session 11 on, all problems were presented in black. No other features of presentation were changed after session 10.

Test examination for verbal production identical to the baseline examination were conducted after phase 1 (T1), before and after phase 2 (T2 and T3, respectively), and about four months after the end of phase 2 (T4). Examinations of keyboard production, testing the effect of colour, were conducted between BL and session 1 of training phase 1 (TA), between session 10 and 11 of phase 1 (TB), and between session 20 of phase 1 and T2 (TC). Tests of colour-digit associations were performed at BL, T1 and T2. The time course of test and training sessions is depicted in Figure 1.

[3] There were only nine colours instead of 10, because in none of either set was the digit 9 the second digit of the solution, given that ties and rule-based problems were excluded from training.

time course

	BL	Baseline verbal	(2 sessions)
	TA	Test keyboard	(1 session)
Phase 1	S 1-5	Training colored presentation + legend	(5 sessions)
Phase 1	S 6-10	Training colored presentation	(5 sessions)
Phase 1	TB	Test keyboard	(1 session)
Phase 1	S 11-20	Training black presentation	(10 sessions)
	TC	Test keyboard	(1 session)
	T1	Test verbal	(2 sessions)
	Pause	no training	(~ 1 month)
	T2	Test verbal	(2 sessions)
Phase 2	S 1-5	Training colored presentation + legend	(5 sessions)
Phase 2	S 6-10	Training colored presentation	(5 sessions)
Phase 2	S 11-18	Training black presentation	(8 sessions)
Phase 2	Pause	illness, no training	(~ 2 weeks)
Phase 2	S 19-25	Training black presentation	(7 sessions)
	T3	Test verbal	(2 sessions)
	Pause	no number-specific training	(~ 4 months)
	T4	Test verbal	(2 sessions)

Figure 1. Time course of the rehabilitation programme.

Results

Effects on trained items

The dependence of mean response latency on time (session) and the kind of help (cue) was described by a stepwise regression model. If no help was offered one would expect an exponential decay of response latency with time (session). Offering additional help from the beginning, which was removed at two points in time (S6 and S11), one might expect an exponential decay of response latency, interrupted at S6 and S11 by an increase of response latency with an exponential decrease following. Therefore, we fitted exponential decreasing models allowing no increase, 1 increase at S11 and 2 increases at S6 and S11. We fitted these models using the more direct way of non-linear regression in SPSS, which showed a slightly better fit than linear regression with the logarithm of mean response latency as independent variable.

The purely exponential decay can explain 25% of the variance ($R^2 = .25$); the Kruskal–Wallis Test shows that there are still differences in the residuals depending on the session: $p < .001$. The model allowing an increase at the removal of the colour help (S11) is much better than the previous one, explaining 34% of the variance ($R^2 = .34$). The Kruskal–Wallis Test shows that there are no differences in the mean residual any more. An additional increase at the removal of the legend (S6) does not have any advantage, also explaining 34% of the variance ($R^2 = .34$). The predictions of resulting models are shown in the Appendix. In this graph it can be seen that the model with an increase at the removal of the colour help (S11) is much better than the purely exponentially decaying model and that an additional increase at the removal of the legend (S6) does not bring any advantage.

During both phases of training ME's performance improved significantly in terms of keyboard response latencies (mean RT phase 1: 6903 ms to 2836 ms; phase 2: 6829 ms to 2034 ms, see Figures 2 and 3; $R = .579$). This improvement was parallelled by a numerical increase of accuracy, excluding speed-accuracy trade-offs (phase 1: 62/70 to 70/70 correct; phase 2: 64/70 to 69/70 correct). In the course of phase 1, there was a temporary numerical increase of mean response latencies and error rates at both points after a cue was removed (S5 to S6: 4624 ms to 5672 ms and 2/70 to 4/70; S10 to S11: 2962 ms to 4189 ms and

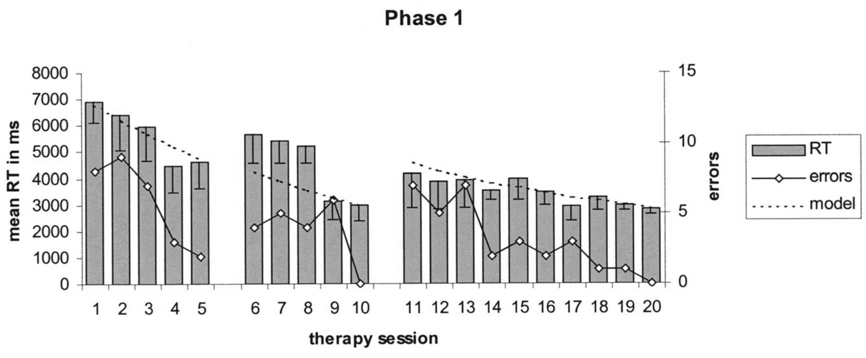

Figure 2. Time course of keyboard multiplication training: Phase 1. Errors are given in absolute numbers; standard deviations indicated on top of each column; model= predicted values of the statistical model describing the dependence of RT on time.

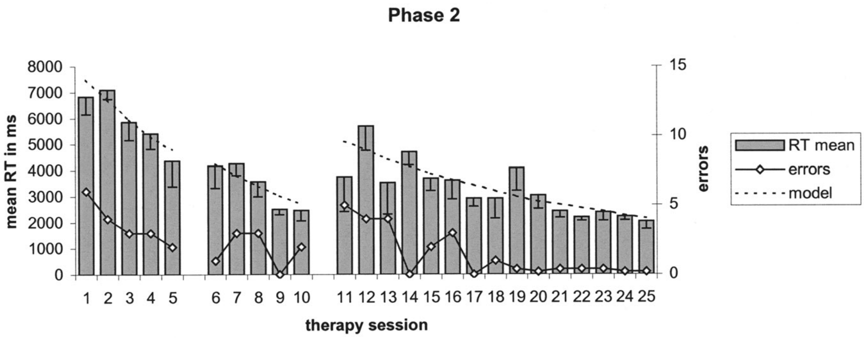

Figure 3. Time course of keyboard multiplication training: Phase 2. Errors are given in absolute numbers; standard deviations indicated on top of each column; model = predicted values of the statistical model describing the dependence of RT on time.

0/70 to7/70). As already mentioned, only the increase of response latencies after removal of coloured presentation contributed significantly to the fit of the statistical model. In the course of phase 2 such increase of response latencies and error rate could only be observed after the second cue was removed (2462 ms to 3752 ms and 2/70 to 5/70). Again, this increase of response latencies contributed significantly to the fit of the statistical model. However, no such increase of response latencies and error rates occurred after removal of the legend in phase 2 (4370 ms to 4180 and 2/70 to 1/70). This is not surprising, given that the patient had already successfully used the colour–digit associations during phase 1 and had the opportunity to relearn them during the first five sessions of phase 2.

Effects of coloured cues

An additional analysis aimed specifically at examining the effects of coloured presentation during phase 1. The same 14 problems of Set A were either uniformly presented in black or in different colours partly indicating the result (as specified above). ME's mean latencies for correct keyboard responses for coloured presentation were compared with RTs for black presentation before training (TA), after the first 10 training session of phase 1 (TB) and after the last session of phase 1 (TC, see Figure 1). As can be seen in Figure 4, there was a general improvement for black as well as for coloured presentation (TA to TC: black: 7901 ms to 3821 ms; coloured: 9580 ms to 3381 ms; Friedman black: $\chi^2 = 10.889$, $p = .004$ $\chi^2 = 20.462$, $p = .000$). Yet, whereas coloured presentation did not help the patient before training, he was significantly faster to respond to the same problems when they were presented with colour cues after 10 sessions of coloured presentation (TB: black: 7150 ms; coloured: 3632 ms; Wilcoxon black vs. coloured: TA: $Z = -0.280$, $p = .779$;

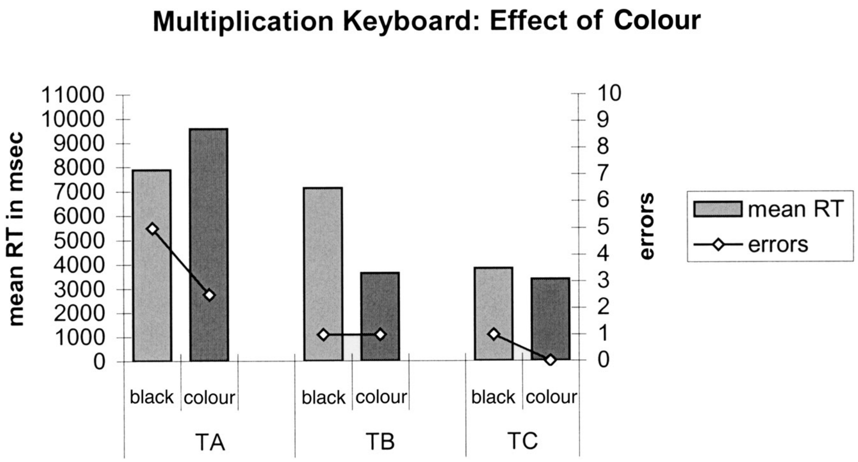

Figure 4. Multiplication keyboard: Response latencies for Set A in black and white vs. coloured presentation. TA = before training, TB = after training session 10, i.e. after the removal of coloured cues, TC = after training phase 1.

TB: $Z = -3.180$, $p = .001$). Crucially, there was a significant improvement for presentation with coloured cues while no such improvement could be observed for black presentation (Wilcoxon TB vs. TA: coloured: $Z = -3.180$, $p = .001$; black: $Z = -0.889$, $p = .374$). This advantage for coloured presentation disappeared after another 10 sessions of black presentation (Wilcoxon black vs. coloured: TC: $Z = -1.013$, $p = .311$). Obviously, the disappearing advantage for coloured presentation at TC was due to an improvement for uncued (i.e., black) presentation rather than due to a deterioration for cued (i.e., coloured) presentation (Wilcoxon TC vs. TB: black: $Z = -2.746$, $p = .006$; coloured: $Z = -1.287$, $p = .198$).[4]

As described in the methods section, the same 14 problems were presented in three blocks (coloured, black, and coloured, respectively). Response latencies for coloured presentation are, in fact, the means of two blocks. To exclude possible order effects. Wilcoxon analyses were performed to compare RTs of blocks one and three (coloured presentation) of each session (TA, TB, and TC). No significant difference was found. In fact, both blocks of coloured presentation were consistently slower (TA) or faster (TB) than the block of black presentation which was administered between them.

[4] Given the number of statistical comparisons performed, a conservative correction of the level of significance (i.e., Bonferroni-correction) was performed for all data of this section. The same p-values as before remained significant.

Comparisons of error rates did not reach statistical significance in any possible comparison.

Digit–colour associations

At all three occasions asked (BL, T1, and T2), ME denied having any spontaneous association between digits and colours. At the latter two occasions (at T1 and T2), he was nevertheless asked to associate colours to single digits. There was no consistency in his answers (1/10 same associations). Furthermore, his answers showed no relationship to the colours used as cues during session 1 to 10 of the first training phase (1/9 and 0/9 "correct" associations, respectively).

Effects on non-trained output-modality

Both in training and baseline/retest sessions problems were presented on a computer screen in Arabic format. While ME was required to give keyboard responses during all 45 training sessions, verbal response were required in the baseline and retest examinations. Nevertheless, he showed a strong improvement in response latencies after training even in this untrained output-modality (mean RTs for multiplication facts BL to T3: 9882 ms to 5369 ms; Wilcoxon $Z = -4.785$, $p = .000$) while his error rate remained virtually the same (1.9% to 2.0%).

Comparison of sets

During phase 1, only the 14 problems of set A were trained while the 14 problems of set B served as controls. Nevertheless, significant improvements in reaction times occurred for both sets between the verbal baseline examination BL and the first verbal retest examination T1 (med RTs set A: 8765 ms to 4273 ms; set B: 6984 ms to 3484 ms; Wilcoxon BL vs. T1 set A: $Z = -2.824$, $p = .005$; set B: $Z = -2.293$, $p = .022$, see Figure 5). Furthermore, there was no significant difference between the improvement rates of set A and set B (Mann-Whitney $Z = -0.659$, $p = .510$).

During phase 2, only the 14 problems of set B were trained while the 14 problems of set A served as controls. Although response latencies decreased numerically for set A as well as for set B between verbal retest examinations T2 and T3 (i.e., before and after phase 2; see Figure 5), this improvement failed to reach statistical significance for both sets (med RTs set A: 4969 ms to 3844 ms; set B: 4789 ms to 4305 ms; Wilcoxon T2 vs. T3 set A: $Z = -0.245$, $p = .805$; set B: $Z = -1.400$, $p = .161$). Again, there was no significant difference between the improvement rates of both sets (Mann-Whitney $Z = -0.301$, $p = .764$).

There was no significant change in error rates for set A or set B during both phase 1 and phase 2 (McNemar all ps $\leq .500$).

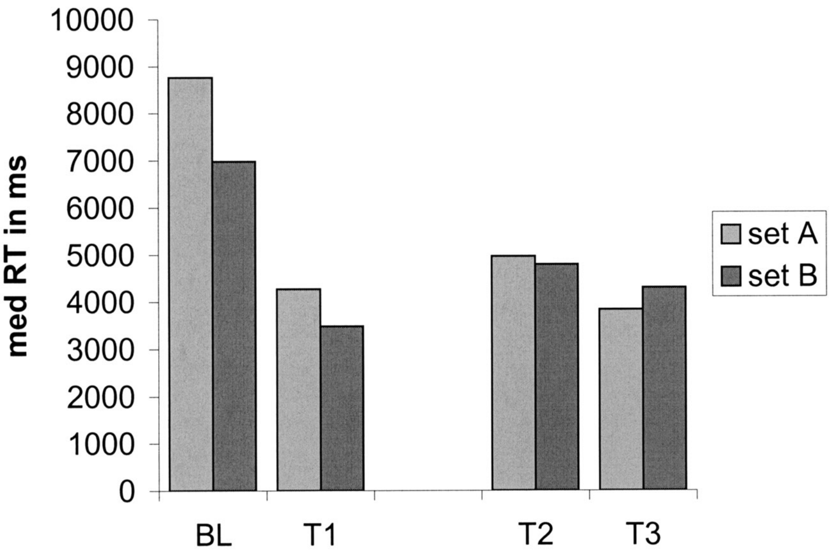

Figure 5. Multiplication verbal: Comparison of sets.

Comparison of operand orders

During both phase 1 and phase 2 of the training, only problems of the format "large times small" (l × s) was presented. Nevertheless, a significant decrease in verbal response latencies could be observed after training not only for (trained) l × s problems but also for their (nontrained) complement problems (med RTs BL to T3: l × s: 7375 to 3844; s × l: 8320 ms to 5532 ms; Wilcoxon BL vs. T3: l × s: $Z = -3.139$, $p = .002$; s × l: $Z = -3.486$, $p = .000$). Furthermore, there was no significant difference between improvement rates of both operand orders (Wilcoxon $Z = -0.544$, $p = .586$). While the training resulted in the above mentioned decrease of response latencies for both operand orders, it left error rates unaffected.

Effects on division performance

Given that division can be regarded as a complementary operation to multiplication, a further analysis aimed at examining possible generalisation effects from multiplication training to ME's division performance. There was a numerical decrease of verbal response latencies for division facts between the BL and T3 (med RTs 8469 ms and 7672 ms). However, this decrease failed to reach statistical significance (Wilcoxon T3 vs. BL: $Z = -1.208$, $p = .227$). Furthermore, there was also a decrease of error rates for division facts (BL: 6/62 = 9.7%; T3: 1/62 = 1.6%), which failed to reach significance, too (McNemar $p = .125$).

When division problems complementary to set A and set B of multiplication problems (e.g., Set A multiplication: 6 × 4, complementary division problems: 24 ÷ 6 and 24 ÷ 4) are analysed separately, no significant difference between sets can be found—either for phase 1 or phase 2 (Mann-Whitney T1 vs. BL: $Z = -0.448$, $p = .654$; T3 vs. T2: $Z = -1.187$, $p = .235$).

Effects on addition and subtraction

To investigate possible generalisation effects of multiplication training to verbal addition and multiplication, the patient's performance was compared for both operations before and after training. No significant change could be observed, either for response latencies or for accuracy (BL to T3: mean RTs addition: 3419 ms to 3769 ms; subtraction: 6135 ms to 6524 ms; error rates addition: 1% to 5%; subtraction 3% to 6%).

Given that repeated addition can potentially be used as a back-up strategy for multiplication (Girelli et al., 1996; Siegler, 1988) a separate analysis was performed for addition ties, as ties are especially involved in repeated addition strategies (e.g., repeated addition for all multiplication problems of the 7 times table involves the addition tie 7 + 7). There was no improvement for addition ties during multiplication training (mean RTs 3148 ms to 3541 ms; error rate in both examinations 0%).

Long-term effects

There was a significant decrease of verbal response latencies for multiplication facts between BL and T3 (mean RT 9882 ms and 5369 ms, respectively; Wilcoxon $Z = -4.785$, $p = .000$). To examine possible long-term effects of training, the same retest examination was repeated after about four months without any specific training (T4). ME's performance remained stable during this period, indicating long-term gains from intervention (mean RT T4: 5384 ms; Wilcoxon T4 vs. T3: $Z = -0.147$, $p = .884$). No significant change for error rates was observed—either between BL and T3 or between T3 and T4 (NcNemar $p \leq .375$).

Standard effects of multiplication fact retrieval

Following suggestions of Whetstone (1998), post-hoc analyses were performed to examine possible size and ties effects. As both kinds of effects (i.e., an advantage for small problems compared to large problems and an advantage for problems with identical operands compared to other problems) are commonly found in healthy adults, their existence in a patient's RT data would, according to Whetstone, point to a qualitatively normal behaviour.

ME showed both a huge advantage for small (product < 27) compared to large (product ≥ 27) problems (difference BL: 6945 ms; T3: 3993 ms) and an advantage for ties compared to other multiplication facts (BL: 2148 ms; T3: 2000 ms). While the former difference proved to be highly significant, the latter failed to reach statistical significance, probably due to the small amount of tie problems ($n = 8$) (Mann-Whitney size effect both differences $Z = -3.480$, $p = .001$; ties effect both differences $p \leq .242$).

Rule-based problems

Problems involving 1 as an operand were answered correctly in 16/17 cases (94.1%) in the baseline testing and without any error in all subsequent sessions (see Figure 6). In contrast, ME showed in the baseline testing 19/20 (95.0%) errors in problems involving 0 ($n \times 0$; $0 \times n$). In fact, only the problem 0×0 was answered correctly. Both problem 0×1 and problem 1×0 were answered wrongly ("1") and analysed as 0 problems. Subsequently, the patient was instructed how to answer this type of problem correctly. The instruction was restricted to the simple explanation that everything times zero gives zero. Although never trained, problems involving zero showed a dramatic increase in accuracy between BL and T1 (T1: 4.8% errors; McNemar $p = .000$). However, this first intervention did not have a long-lasting effect. While accuracy did not change significantly from T1 to T2, it decreased again from T2 to T3 (10.5% to 95.2% errors). After the conclusion of T3, the 0 rule was repeated and explained to the patient ("anything times zero gives zero"). Again, error rates dropped substantially and then remained stable for at least four months, as can be seen in the fourth testing session (see Figure 6) (T4: 0% errors, McNemar $p = .000$).

Correctly solved rule-based problems yielded significantly faster RTs than correctly solved multiplication facts—both before and after training (mean

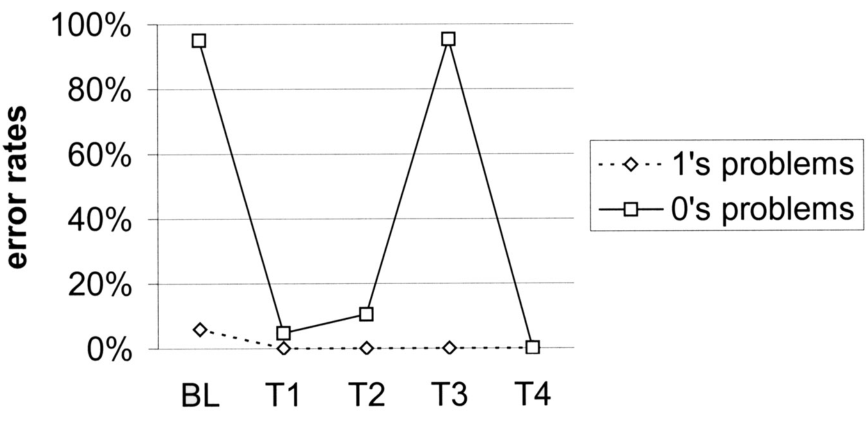

Figure 6. Multiplication verbal: Rule-based problems.

RTs BL: rules 2403.4 ms, facts 9881.8 ms; T3: rules 1737.3 ms, facts 5368.7 ms; Mann-Whitney BL: $Z = -5.973, p = .000$; T3: $Z = -6.902, p = .000$). Within the set of rule-based problems no analysis of reaction times could be performed due to the low number of correct items in two of the five testing sessions.

Attention

A retest examination of alertness and divided attention was performed after training phase 2 to disentangle possible gains specifically due to the trained operation from generally improved attention. Neither alertness nor divided attention improved. In fact, the patient's performance was numerically worse for both measures of attention after the training of fact retrieval (median RT alertness: 1197 ms to 1556 ms, divided attention: 1344 ms to 1692 ms).[5]

DISCUSSION

We reported a successful and long-lasting remediation of the retrieval of simple multiplication facts in ME, a patient in the chronic stage after traumatic brain injury. Given that ME already performed in the normal range before rehabilitation in terms of error rates, the main target of the intervention was to reduce his excessive response latencies. After extensive repetition of a subset of multiplication facts on a computer keyboard, the patient reached substantially decreased response latencies. On the one hand, these improvements generalised not only to a non-trained operand-order ($s \times l$), but also to a non-trained output-modality (verbal), and even to a non-trained set of multiplication problems (set B in phase 1). On the other hand, however, these gains turned out to be operation specific: No transfer could be observed for simple addition or subtraction, while the results are less clear for division. Crucially, the coloured presentation of problems facilitated the patient's retrieval of simple multiplication facts, due to successfully established associations between colours and digits.

ME showed generalisation to the non-trained operand order. Similar results have been reported for patients MC (Whetstone, 1998) and JG (McCloskey et al., 1991) while there was no significant generalisation to complement problems found in patient BE (Hittmair-Delazer et al., 1994). Transfer to the complementary operand order for simple multiplication facts in training studies with healthy adults has been reported by Fendrich, Healy, and Bourne (1993) and Rickard, Healy, and Bourne (1994), and priming for complement

[5] A subsequent training targeted to improve attention and fast word recognition in reading resulted in a significant improvement of alertness (median RT 590 ms). Yet, even then ME's performance remained below the 1st percentile.

problems was observed by Delazer, Ewen, and Benke (1997) for amnesic patients and a group of controls. However, priming was not found in every single control subject, pointing to the possibility of individual differences (see also Lefevre et al., 1996).

Some authors argue that division can be regarded as a complementary operation to multiplication. Indeed, ME, as well as BE (Hittmair-Delazer et al., 1994), TL, and ZA (Girelli et al., 1996) showed some improvement for division problems after multiplication training although in the case of ME this effect was too weak to reach statistical significance for response latencies or for error rates. However, the case of patient CB (Cipolotti & de Lacy, 1995), showing an isolated impairment for division (multiplication being preserved), points to a far reaching autonomy of both operations. Studies with healthy subjects, too, yielded somewhat conflicting results concerning the relationship between multiplication and division. Campbell (1997) reported highly correlated RTs and error characteristics for multiplication and division as well as priming of multiplication errors by previous division trials, compatible with the notion of multiplication at least used to check division. Similar results are described by Lefevre and Morris (1999), who found closely related error and latency patterns and cross-operational facilitation by complementary problems (more so from division to multiplication). Moreover, on large division problems, participants reported that they "recast" problems as multiplication. These findings are taken to support the hypothesis that multiplication and division are stored in separate mental representations but that solution of difficult division problems sometimes involves access to multiplication. On the other hand, little if any transfer from multiplication training to division was observed by Rickard et al. (1994, but see Campbell, 1999). Again, individual differences may play some role in a unifying interpretation of these data, as discussed, for example, by Rickard et al. (1994).

As concerns the other basic operations of arithmetic, the gains of the present therapeutic intervention for multiplication did not spread to addition or subtraction. This was not due to a ceiling effect as ME showed excessive response latencies for both operations. Such a pattern clearly supports the view that multiplication is supported by systems different from those of other arithmetic operations. It is compatible with the assumption that the arithmetical fact store is segregated by operations (Dagenbach & McCloskey, 1992; van Harskamp & Cipolotti, 2001; see Ashcraft, 1982, for conflicting evidence).

ME's improvement in performance reached in keyboard training generalised to the verbal output modality. This transfer can be taken as evidence that the observed effect of training was not simply an effect of faster answer execution in the trained modality (i.e., faster pressing of the keys). Encoding, retrieval, and production are three different steps in solving a multiplication problem. Whetstone (1998) emphasised the *match effect* found in his patient MC, i.e., an RT advantage for the trained input format. However, no data are provided concerning possible cross-modal gains. As the output format was always the same (spoken verbal production), the observed training effects may be due to improvements both at the encoding and at the retrieval stage. As Arabic format was used as input for both keyboard and verbal production in our study, the transfer to a non-trained output modality observed for ME most likely has to be addressed to the retrieval stage itself. However, the format in which the patient retrieves multiplication facts cannot be disentangled with the present data. Retrieval might have relied on the verbal format (Dehaene & Cohen, 1995; 1997), on an abstract format (McCloskey, 1992), or on some individually preferred format (Campbell & Clark, 1988; Deloche & Willmes, 2000). Furthermore, we cannot exclude some modality-specific *output match effect* (i.e., larger effects for keyboard compared to verbal output) due to extensive training in only one modality (keyboard). However, this output match effect can be expected to be weak compared to the substantial cross-modal transfer.

Does ME retrieve multiplication facts from memory or does he use fast strategies/procedures? Clearly, the patient's reaction times even after training are still much above the range of fast and automatic retrieval found in healthy adults (mean RTs of less than one second per problem, e.g., Campbell, 1987b). For this reason we cannot fully exclude a speeded use of strategies and procedures by the patient. However, ME was slowed down in general, as evidenced by his poor alertness measures. Furthermore, "fact retrieval" has been claimed in previous studies for different subjects showing excessive RTs for different reasons: For instance, trials with mean RTs of 5.5 s were classified as "retrieval" in third graders if they showed no overt use of strategies (Siegler, 1988). This is in accordance with the author's model which assumes that a search length parameter is to be set before each individual retrieval process, allowing for different (and possibly excessive) latencies in retrieval. Fact "retrieval" has also been assumed for patients BE (Hittmair-Delazer et al., 1994) and MC (Whetstone, 1998), despite mean RTs of up to 6.3 s (BE, pointing to a number table) or between 3.6 s and 6.1 s (MC, spoken verbal input, match condition and written verbal input, nonmatch condition). Whetstone (1998) argues that MC carries out fact retrieval in the same manner as healthy individuals because he displays "effects commonly found in the multiplication fact retrieval performance of non-brain-damaged participants", including the size effect. However, especially the size effect might reflect the use of strategies/procedures even in healthy adults—at least for larger or more difficult problems. In fact, some authors point out that educated healthy adults also partly or exclusively use strategies or procedures to solve simple multiplication tasks (Baroody, 1994; Lefevre et al., 1996). ME's unexpected transfer to non-trained problems might be a hint to a use of strategies as strategies are flexible and can easily be applied to untrained problems. Furthermore, healthy subjects show a reduced or even negative transfer for non-trained facts after training of fact retrieval (Campbell, 1987a; Rickard et al., 1994). This explanation, then, would also hold for ME's improvements for the non-trained operand order and his (nonsignificant) improvement for the complementary operation (i.e., division). Indeed, the patient reported using strategies for some problems even after finishing the training, but he had stopped doing it overtly. Yet, there are arguments against the use of repeated addition as a procedure to solve multiplication problems, because addition ties, which are crucial in this procedure, did not show any improvement. More crucially, the facilitating effect of colour cannot be explained in terms of strategies or procedures but has to be taken as a strong indication of retrieval processes being active in ME.

Two results are noteworthy regarding rule-based problems. First, error rates in answering 0 problems were dramatically higher than both error rates in answering 1 problems and error rates for $n \times$ m problems. Moreover, all errors consisted in the typical $n \times 0 = n$ or $0 \times n = n$ error. The findings thus mesh well with those of McCloskey et al. (1991) who propose separate storage of 0 rules. Second, effects of intervention (i.e., simple statement of the rule) were immediate and clear-cut. While the first explanation did not show a long-lasting effect, the second explanation seemed to have a long-lasting effect as shown in the follow-up test. In contrast to the remediation of $n \times$ m facts, which require extensive and individual training (Girelli et al., 1996; Kashiwagi et al., 1987; Miceli & Capasso, 1991; Whetstone, 1998), rule-based errors may be critically influenced by explanation (see also McCloskey et al., 1991). These findings, again, converge with the assumption that only $n \times$ m problems are stored and retrieved individually.

The use of colour as a visual cue (vanishing in two steps) proved to enhance ME's multiplication performance. At which level might this beneficial effect be located? It can be excluded that the coloured presentation of multiplication problems simply leads to a nonspecific increase in attention due to the saliency of colour (Davidoff, 1991). In this case, the patient should already have profited at the first occasion of testing (TA). The same argument applies to a possible sensory facilitation component. Therefore it can be concluded that coloured presentation improved learning and/or retrieval more directly. Providing colour information and a legend may have reduced the number of errors produced, thus being an instance of the errorless learning approach. More specifically it may have led to peaked associations between problems and correct answers, reducing the probability of concurrent results to be produced (Siegler, 1988). In addition, colour may have directly supported retrieval, providing an additional link to a different memory domain (i.e., colour memory). In this case, it would parallel the use of mnemonic aids in the learning of healthy subjects and the rehabilitation of amnesic patients, where it has frequently been shown that multiple encoding improves performance, e.g., for movements associated to a dialogue to be learnt (Noice & Noice, 2001) or for the use of visual imagery in learning and retrieving words (e.g., Wilson, 1987).

However, it remains unclear whether the kind of errorless learning performed by ME involved explicit or implicit memory functions or both (e.g., Hunkin & Parkin, 1995). The contribution of these different memory functions cannot be disentangled by the present data. Although the meaning of the colours was explained to the patient at the beginning of the training and he was encouraged to actively associate presented cues (i.e., colours) and answers, he was virtually unable to correctly link colours with their associated meaning explicitly—even after successful training with coloured presentation (TB). This seems to indicate an implicit use of this feature by the patient in a variant of a vanishing cues intervention. Yet, it is possible that the test administered to examine ME's explicit knowledge about digit–colour associations was insufficient, because it asked only for mappings between single digits and colours and not between whole problems and colours.[6] Furthermore, ongoing research should address the question, whether colour can facilitate the retrieval of arithmetic facts due to the specific relationship between colours and digits which was also found in healthy subjects (Dehaene, 1997; Galton, 1880a, 1880b; Seron et al., 1992) or whether the same facilitation can be obtained using different mnemonic aids. Finally, it might turn out that linking colour to the second digit of the result is especially effective in languages like German or Dutch, given that these languages show a unit-decade-inversion of verbal numerals (e.g., 21 = *einundzwanzig* [one and twenty]) and the verbal modality has been assumed to play a particular role in the retrieval of simple arithmetic facts (e.g., Dehaene & Cohen, 1995). Again, further research is needed to examine this possibility.

In sum, repeated exposure to the problems and instant feedback on the given solutions was able to durably (re-)strengthen the associations between problem and answer. Coloured cues may have helped in two ways: First, they could have reduced the risk of errors occurring and thus prevented wrong associations being established or strengthened. Second, they could have directly supported activation of the correct result. Further research should address the question in which circumstances colour or additional stimuli can be effectively used as implicit or explicit cues and in which domains this is promising.

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Manuscript received March 2003
Revised manuscript received August 2003

APPENDIX

Boxplots of mean response latencies in phase 1 superimposed by three nonlinear regression models. The model allowing one increase at S11 can explain more of the variance (34%) than the purely exponential (25%). An additional increase at S6 does not have any advantage (34%).